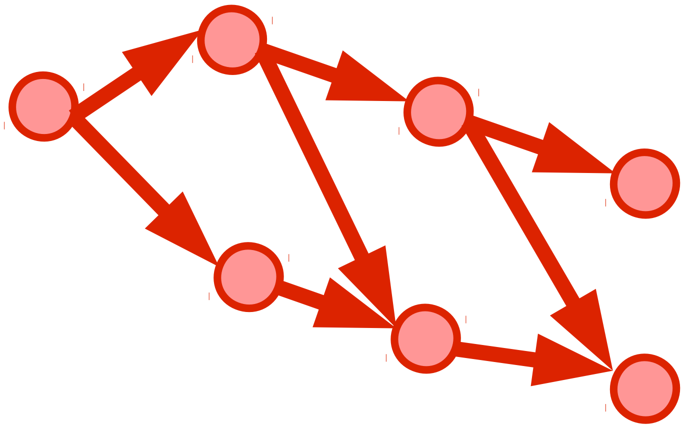

A critical study of network models for neural networks

Gordon Govan, Peirluigi Frisco, Alex Xenos
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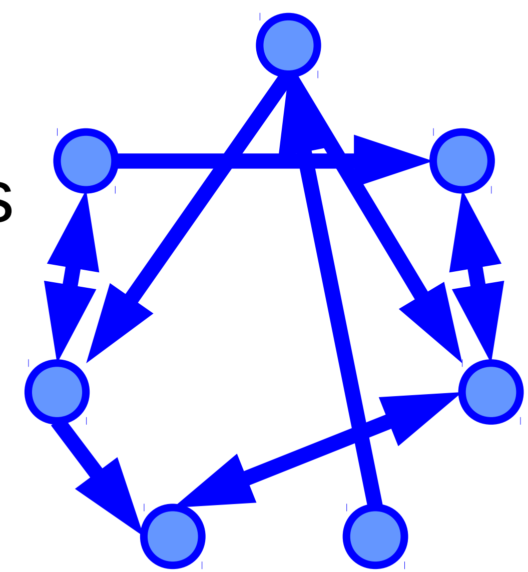
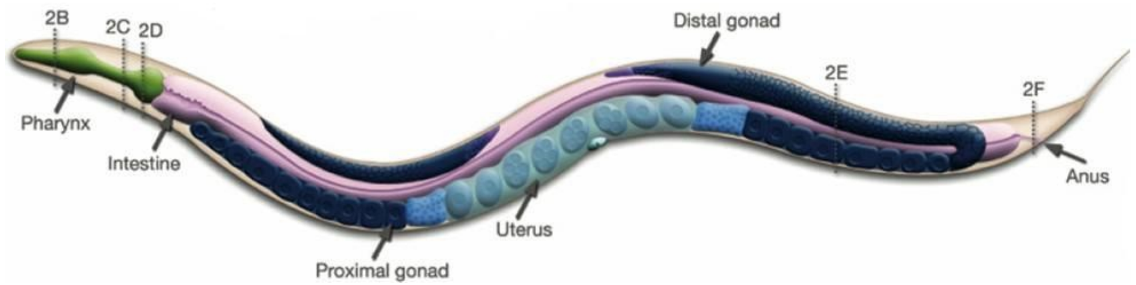


Mathematics of Networks 11 20/07/2012

Abstract

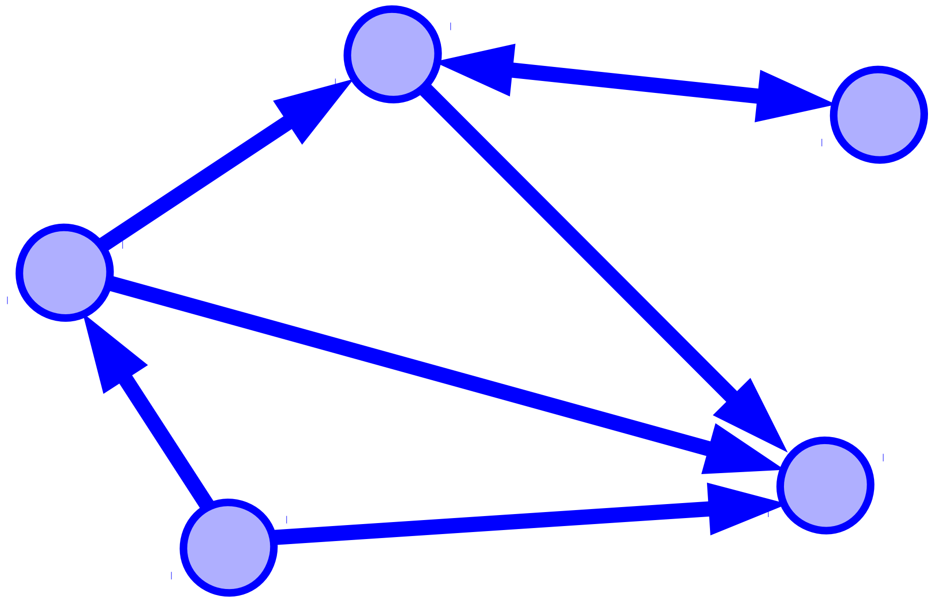
We used different network models to attempt to create networks with similar measurements to the *C. elegans* neural network.



http://publications.csail.mit.edu/abstracts/abstracts07/patrycja/patrycja.html

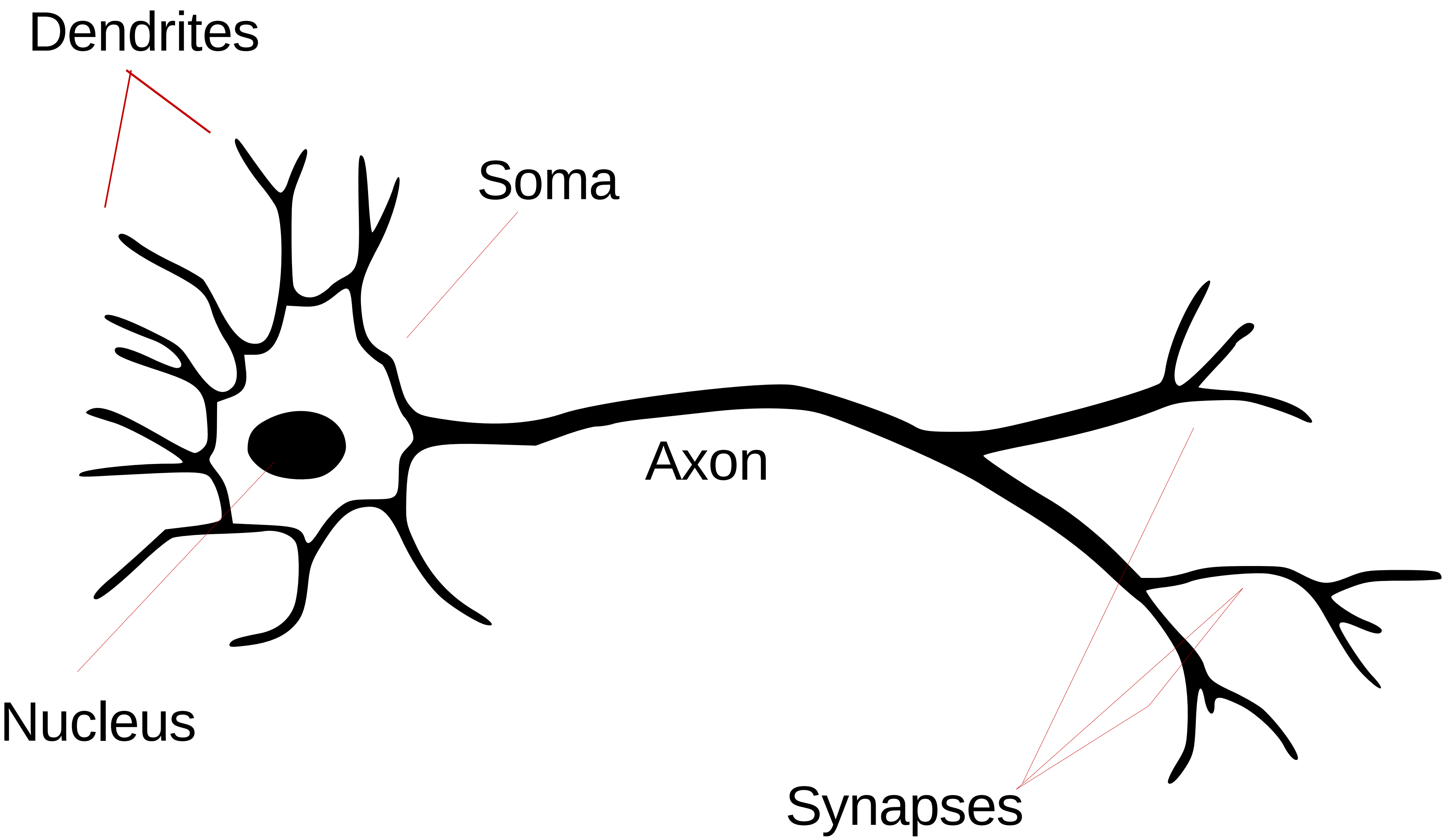
Introduction

many biological concepts can be though of as **networks**



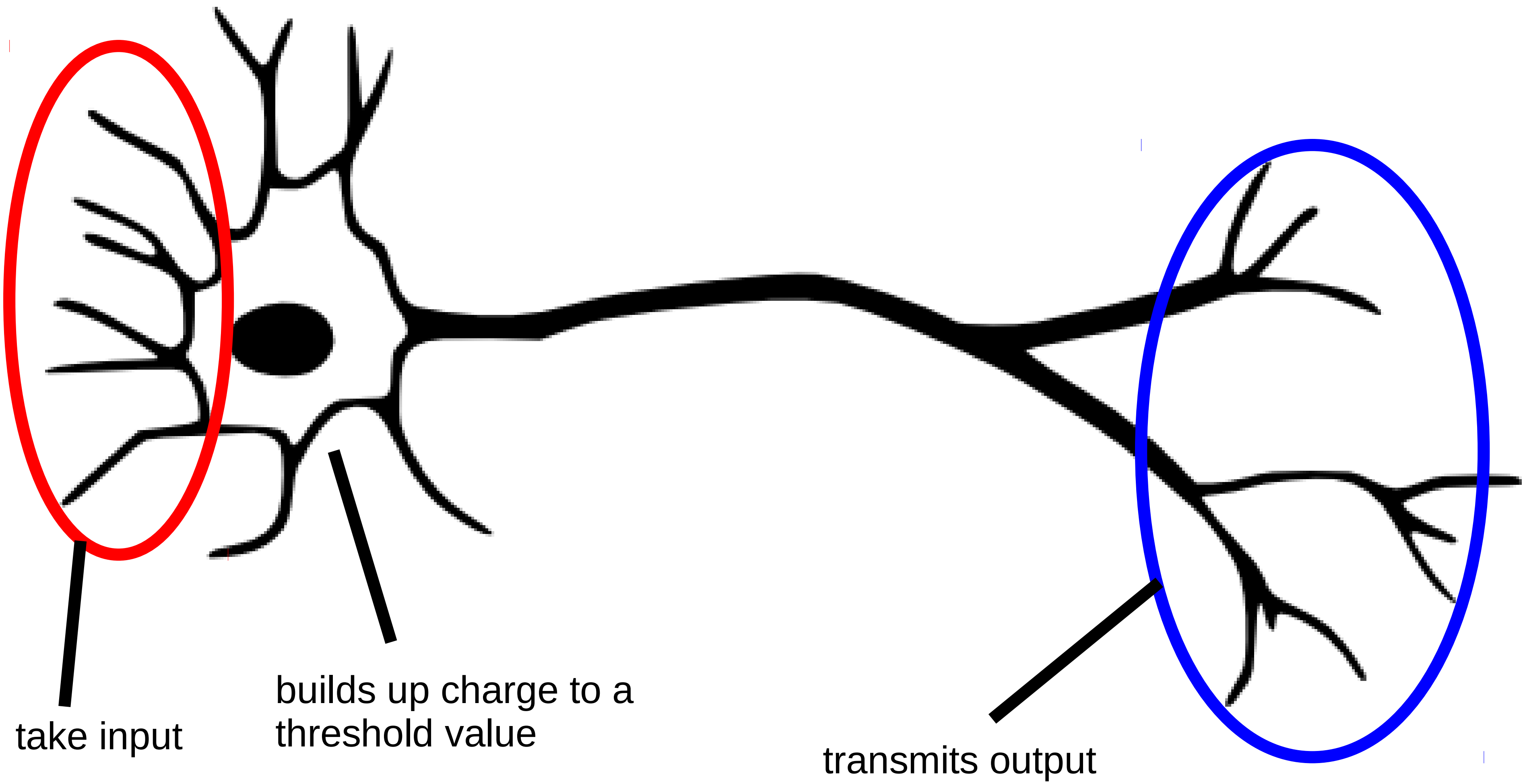
network models have been developed to aid understanding of how these networks develop and operate

Neurons



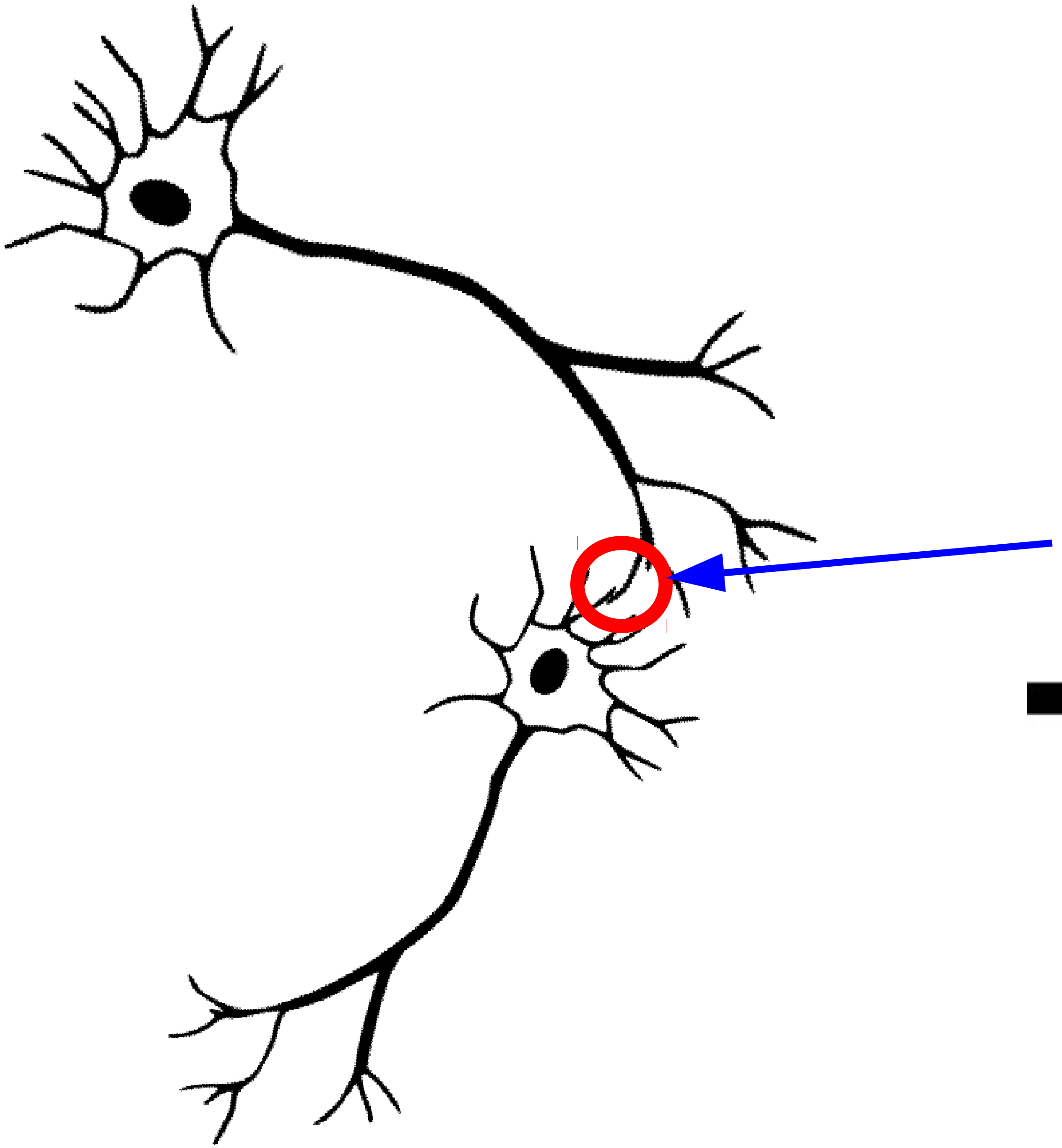
http://upload.wikimedia.org/wikipedia/commons/5/52/Neuron_-_annotated.svg

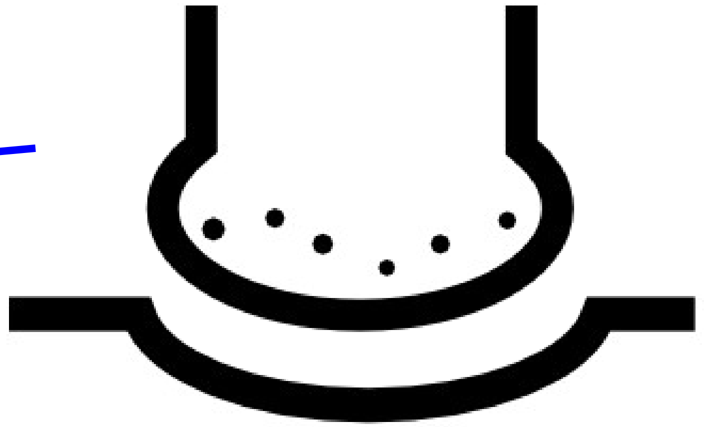
Neurons



http://upload.wikimedia.org/wikipedia/commons/5/52/Neuron_-_annotated.svg

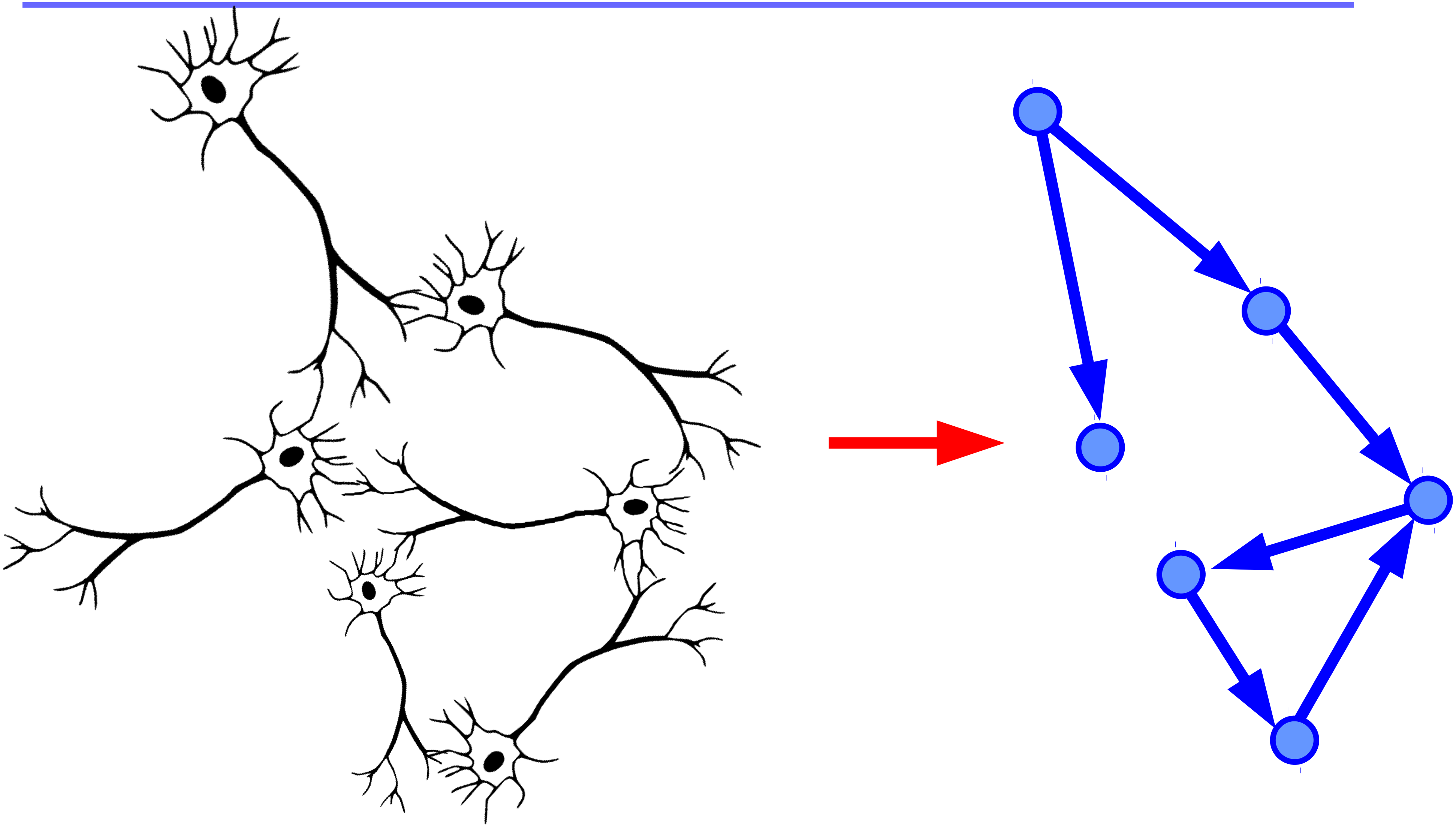
Synaptic junctions





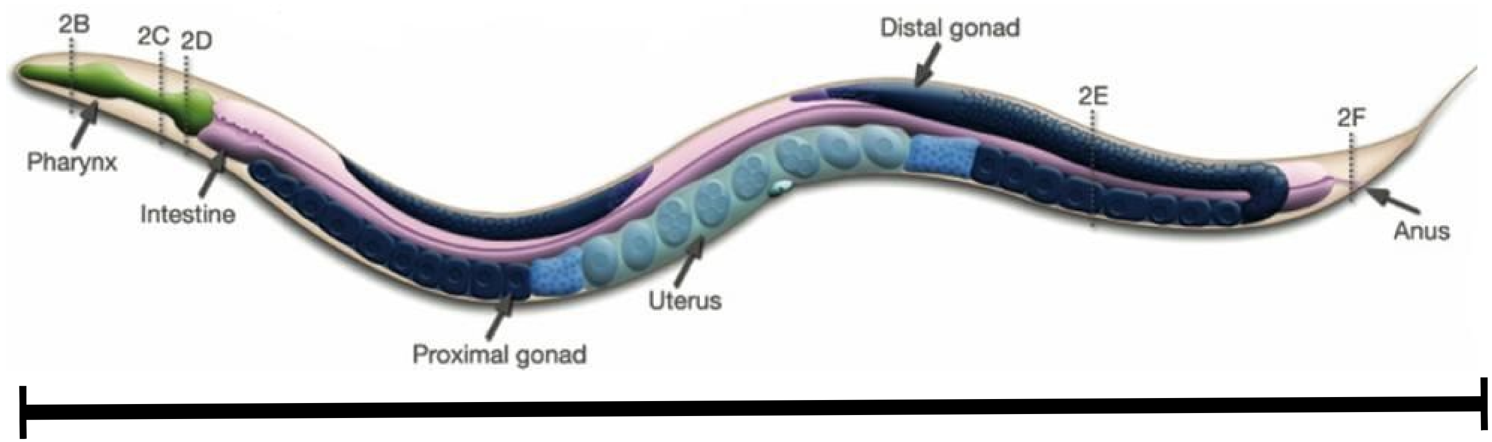
http://upload.wikimedia.org/wikipedia/commons/5/52/Neuron_-_annotated.svg

Neural networks



http://upload.wikimedia.org/wikipedia/commons/5/52/Neuron_-_annotated.svg

Caenorhabditis elegans



Nematode worm

Human brain

Model organism with

306 neurons

2345 synaptic connections

10^{11} neurons

10^{14} synaptic connections

http://publications.csail.mit.edu/abstracts/abstracts07/patrycja/patrycja.html

Network Models

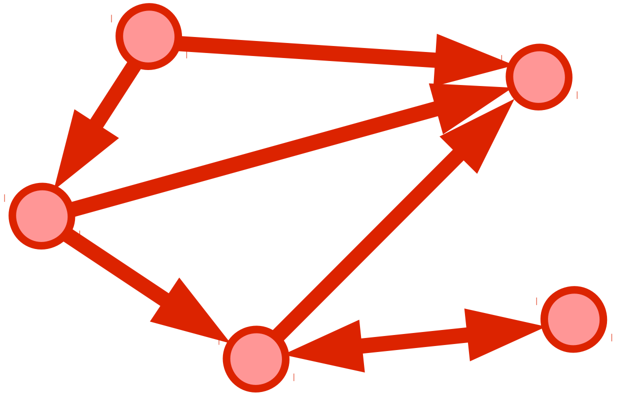
We attempt to create networks with properties similar to that of the *C. elegans* neural network.

To do that we use **network models.**

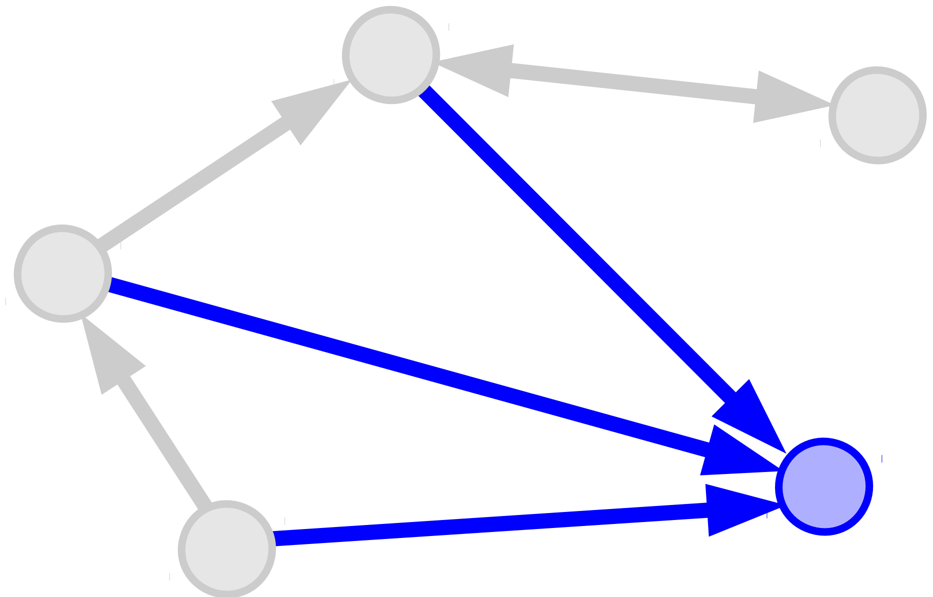
A network model is an algorithm that produces a network from a given input.

randomNetwork :: Int -> Double -> ([Node] , [Edge])

randomNetwork 5 1.75



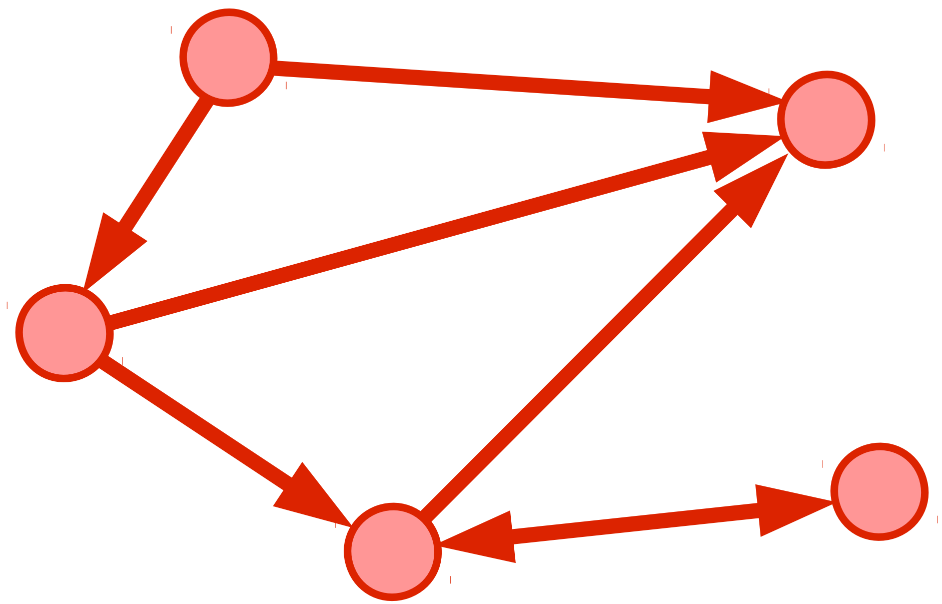
Global topological measurements



306

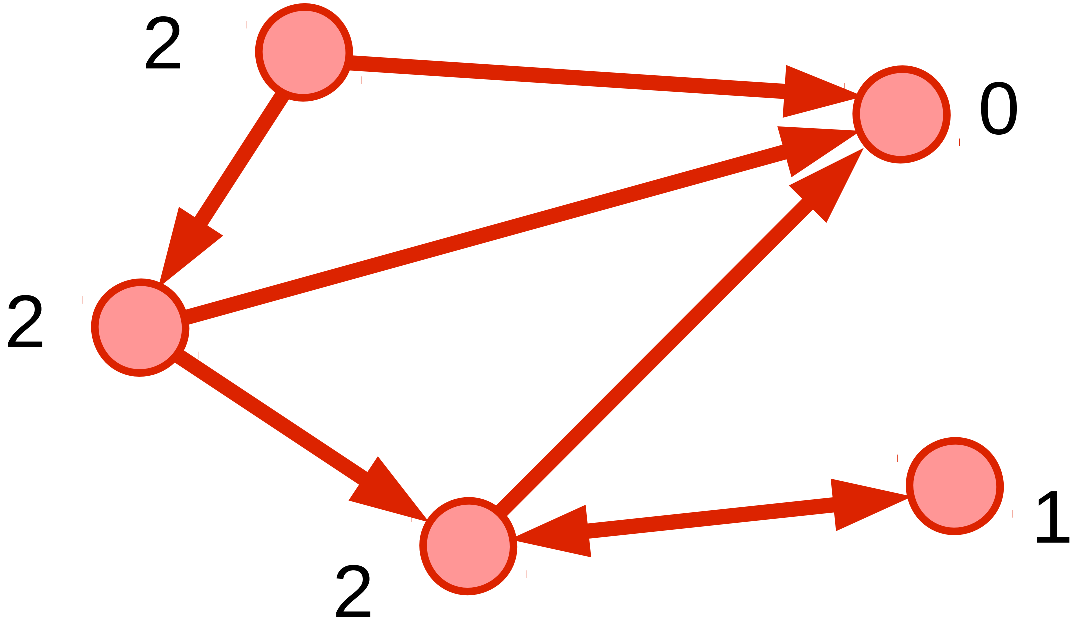
Measurements that we take from each node.
We then take the average from across the network.

Average Degree



the average number of edges that a node in the network has
or
the total number of edges, divided by the number of nodes

Average Degree

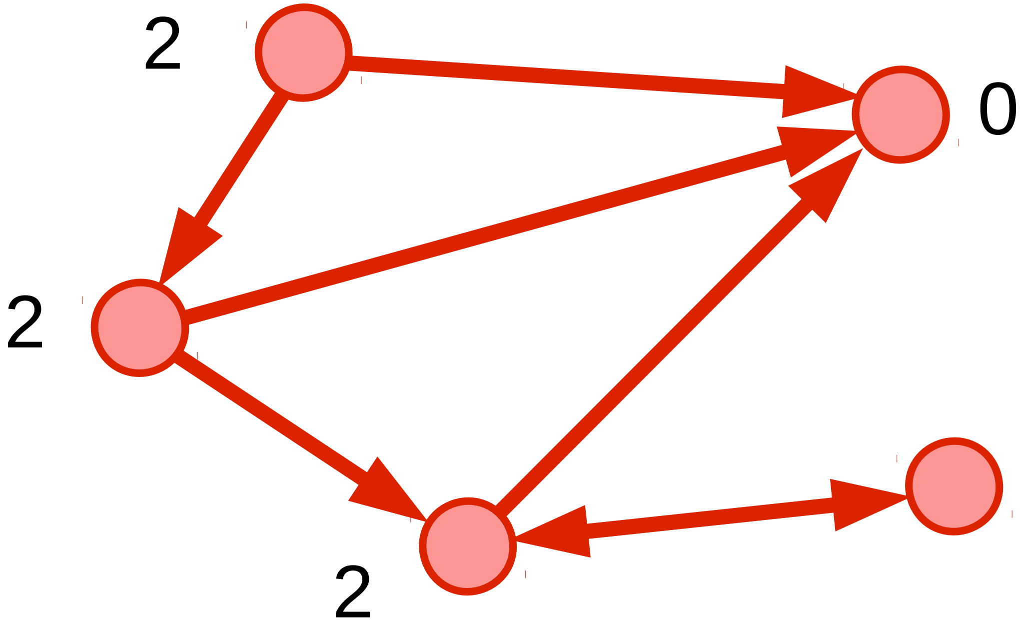


the average number of edges that a node in the network has

or

the total number of edges, divided by the number of nodes

Average Degree



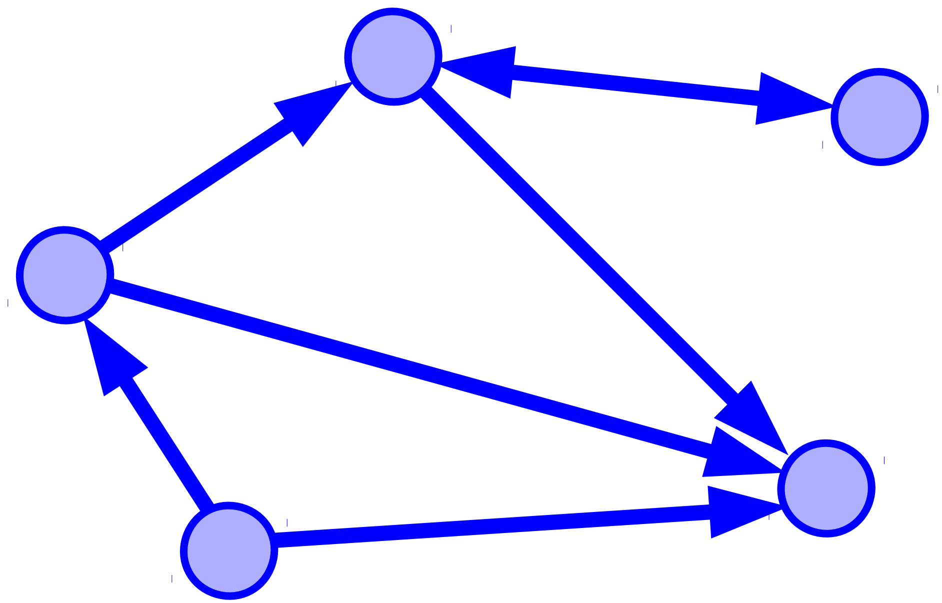
2+2+2+0+1
=7
/5
=1.4

the average number of edges that a node in the network has

or

the total number of edges, divided by the number of nodes

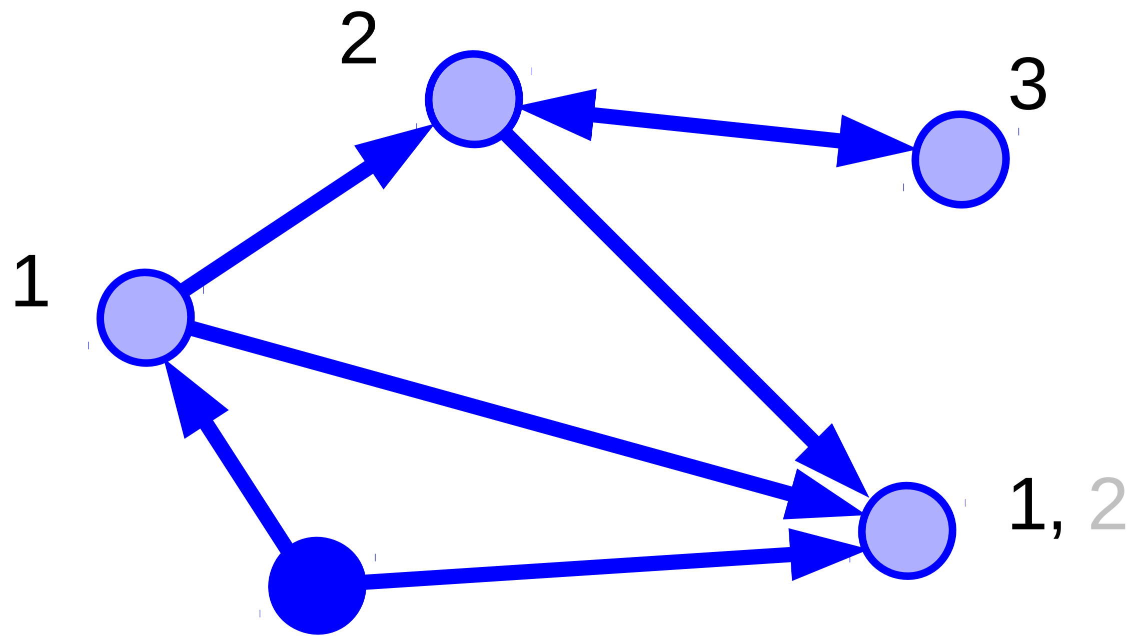
Average Path Length



the average shortest path length between two nodes

calculate the total length of all the shortest paths in the network, and then divide it by the number of paths

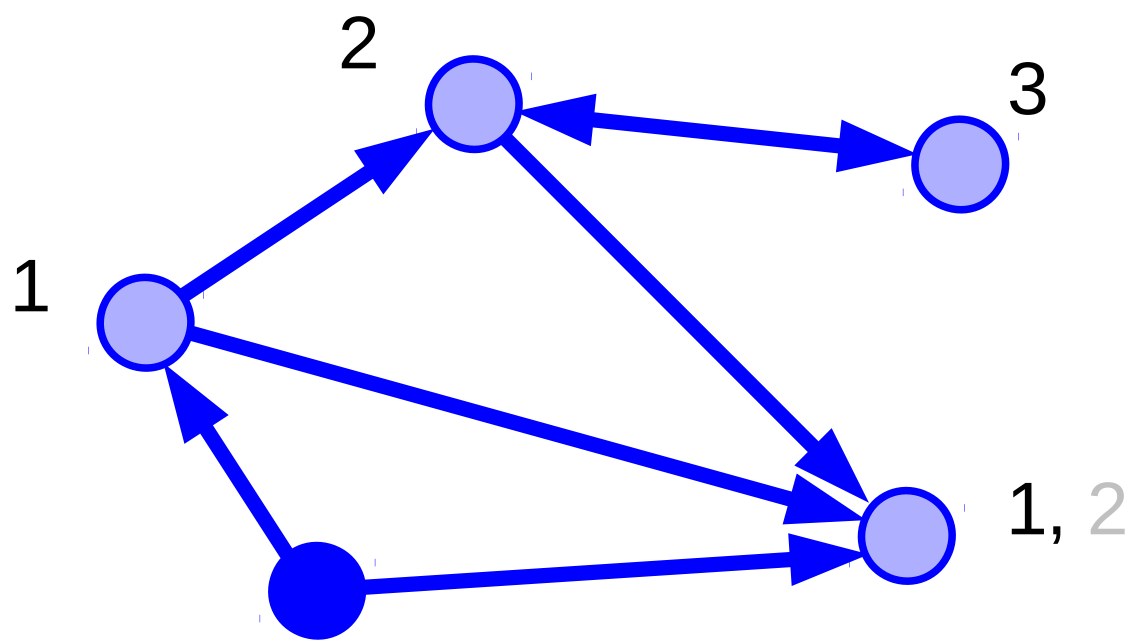
Average Path Length



the average shortest path length between two nodes

calculate the total length of all the shortest paths in the network, and then divide it by the number of paths

Average Path Length

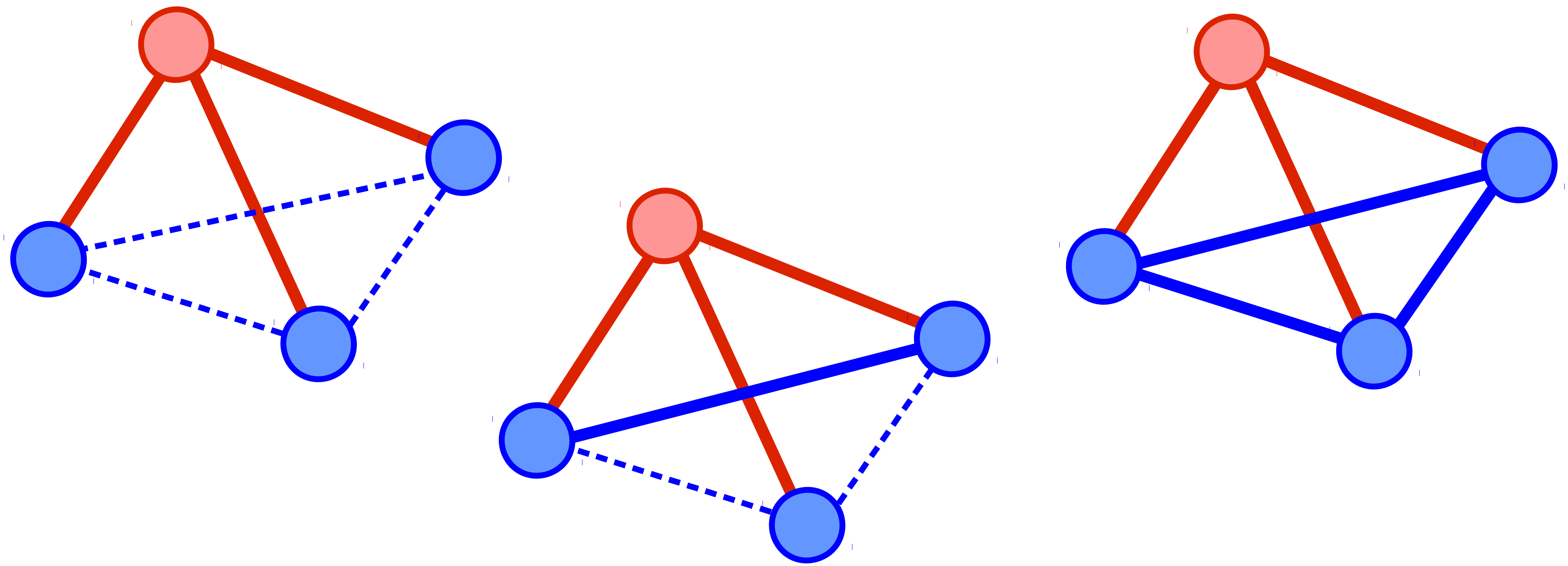


1+1+2+3+1+1
+2+1+1+1+2
=16
/11
=1.45

the average shortest path length between two nodes

calculate the total length of all the shortest paths in the network, and then divide it by the number of paths

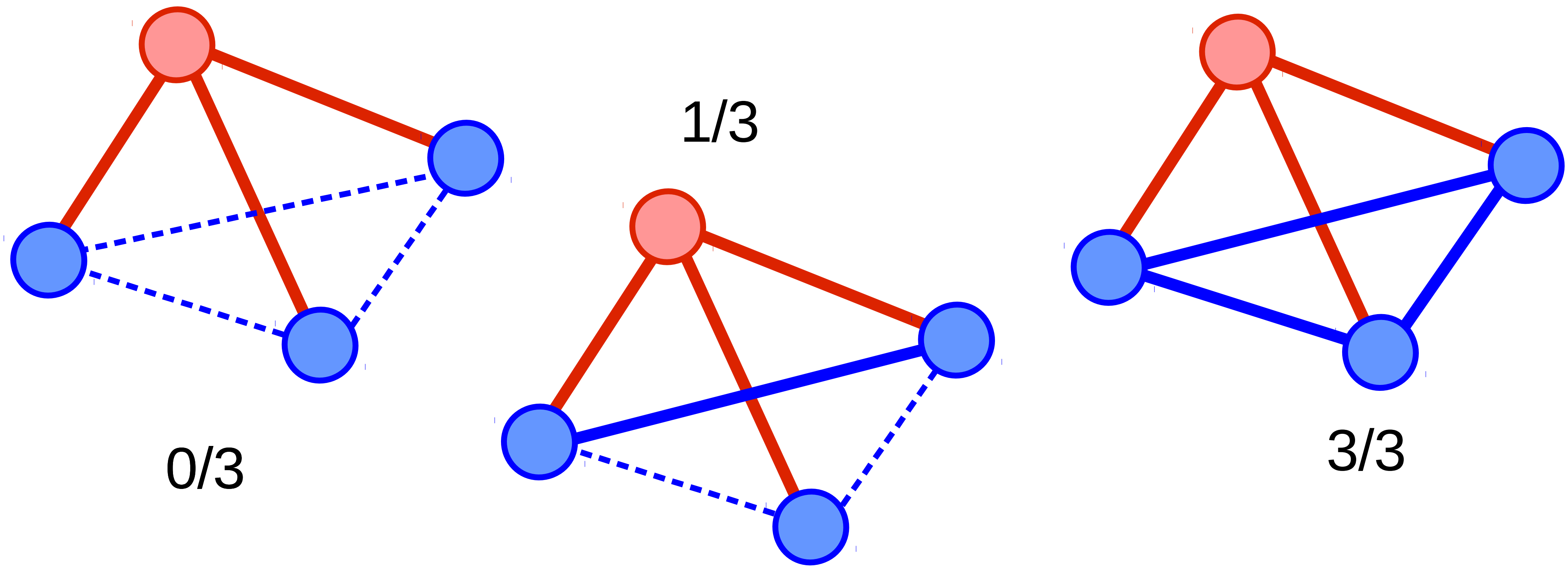
Average Cluster Coefficient



a measure of how many of a node's neighbours are connected together

$$\frac{\text{number of connections between neighbours}}{\text{number of possible connections between neighbours}}$$

Average Cluster Coefficient

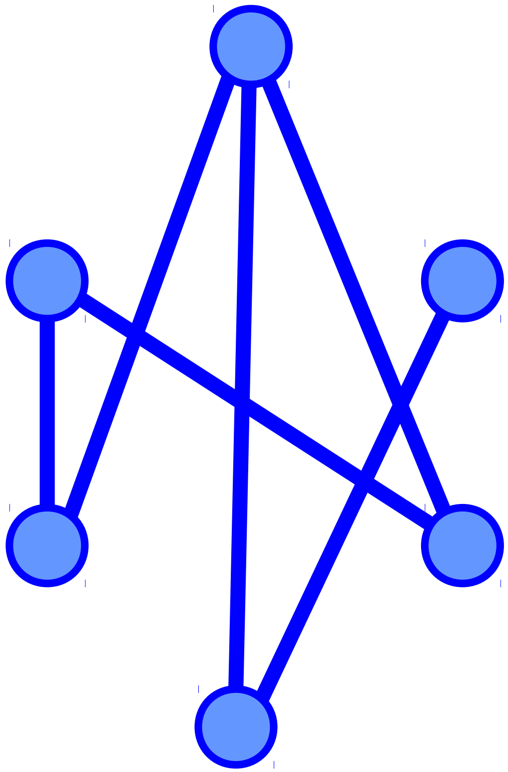


a measure of how many of a node's neighbours are connected together

$$\frac{\text{number of connections between neighbours}}{\text{number of possible connections between neighbours}}$$

Erdős-Rényi model

nodes connected together randomly

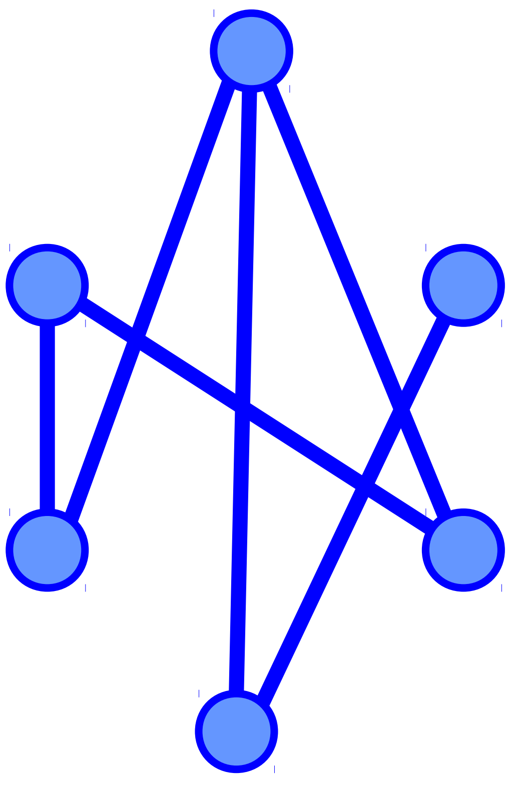


one parameter: probability to have an edge between two nodes

average degree and path length close to *C. elegans*

clustering coefficient much too low

Erdős-Rényi model



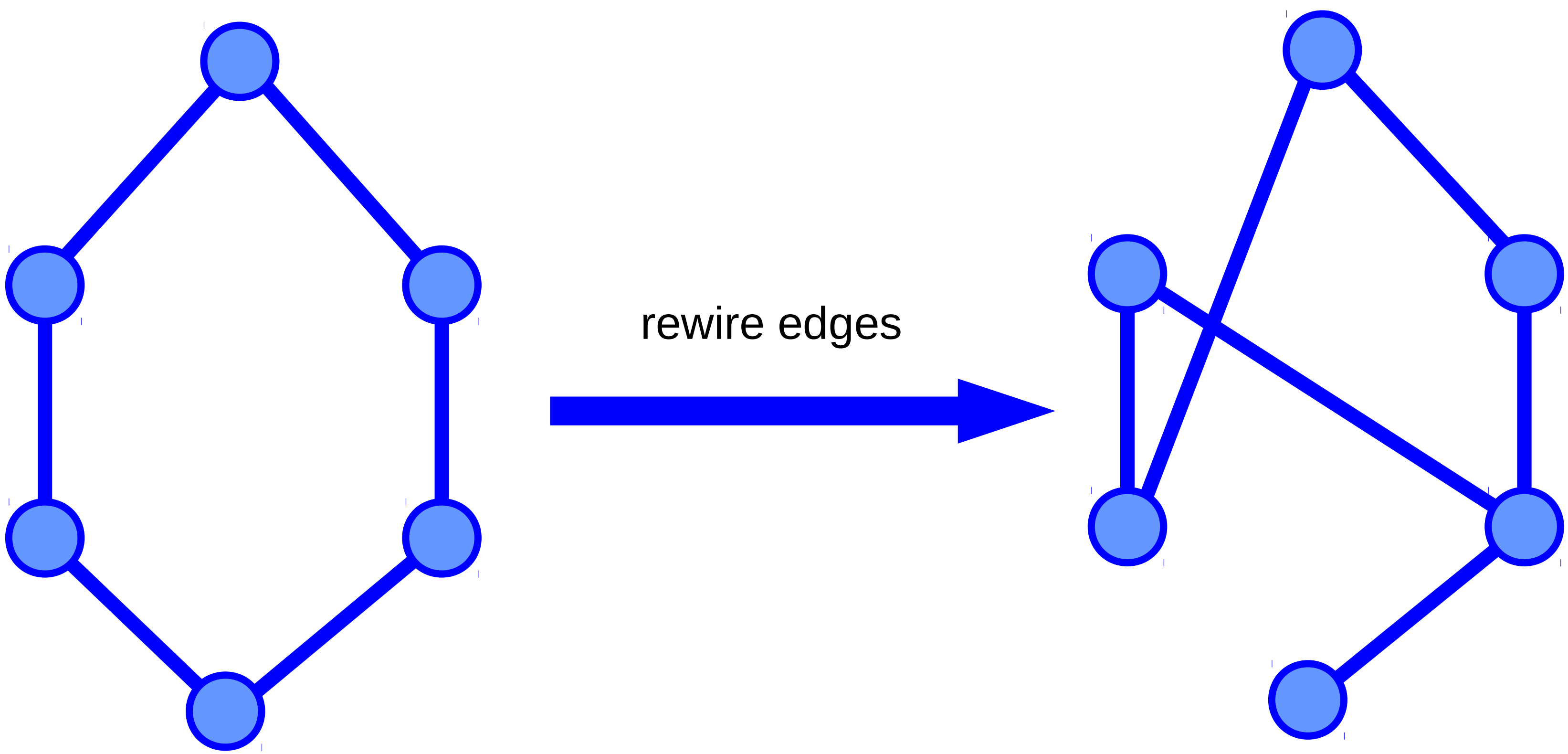
to create the ER networks:

create 306 nodes, connect them with a connection probability of

(avg. degree)/(nodes -1)

Watts-Strogatz model

start with a regular network, then add randomness



Ring Lattice

Watt-Strogatz

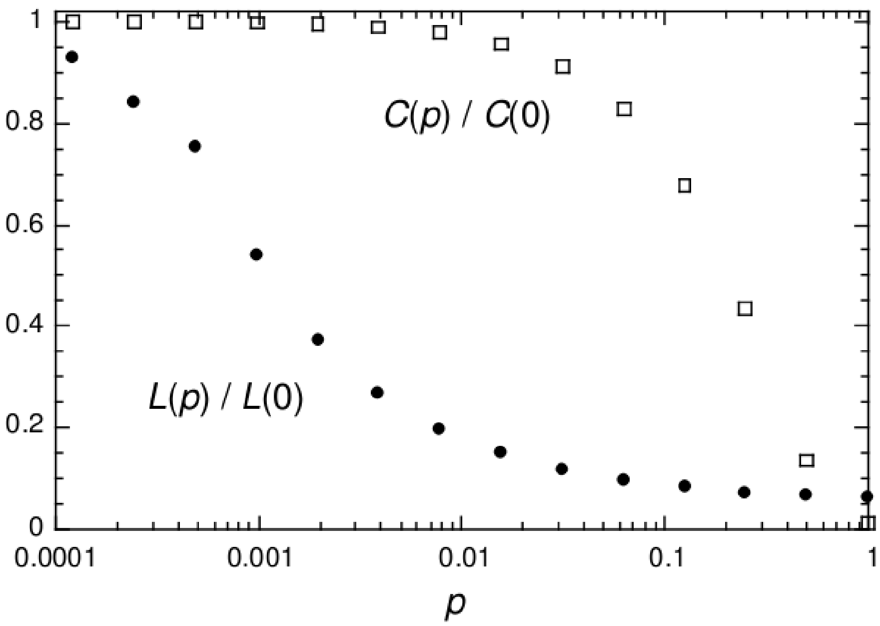
Watts-Strogatz model

two parameters: number of edges, probability to rewire an edge

average degree and path length close to *C. elegans*

and clustering coefficient close to target

Watts-Strogatz model



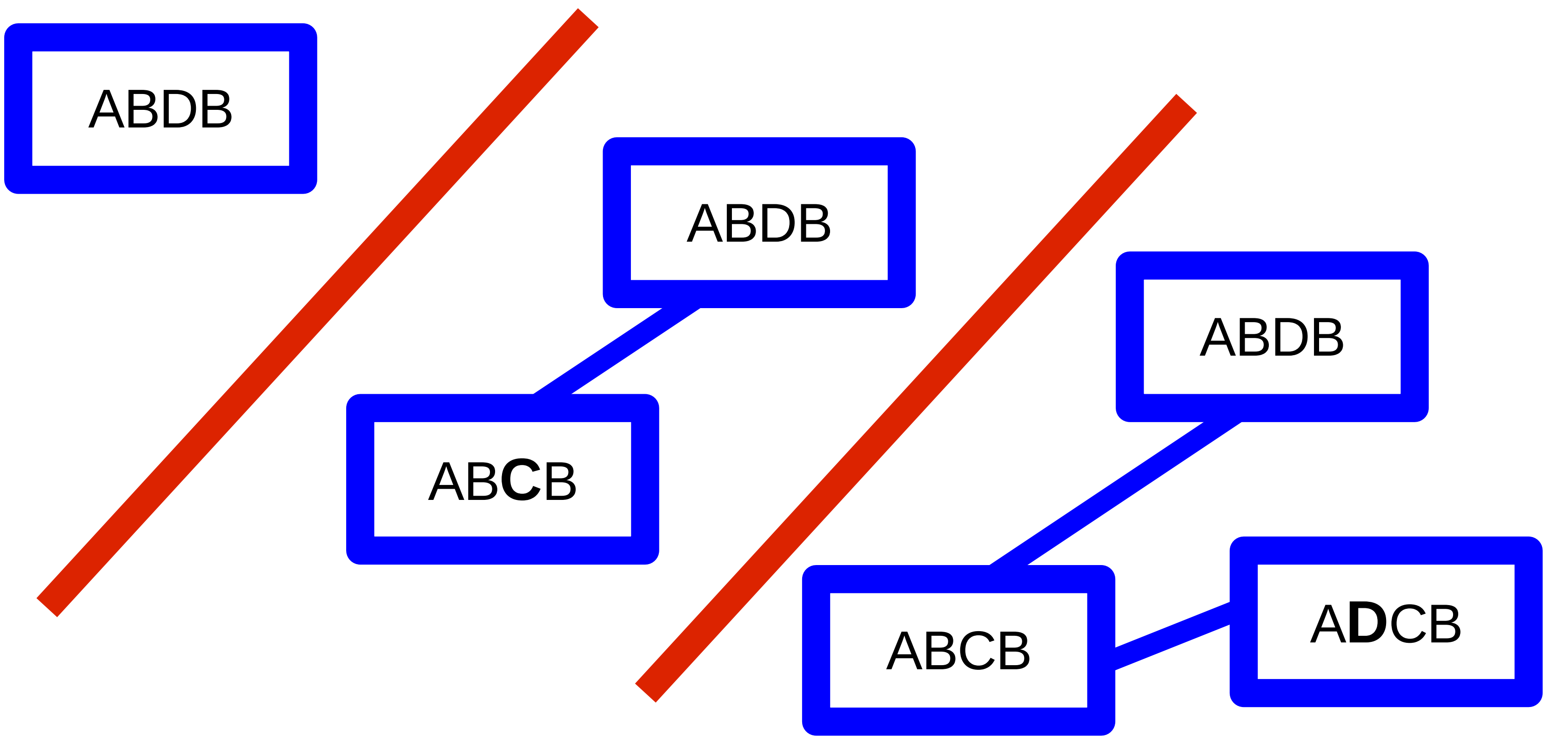
to create the WS networks:

create regular lattices, with each node connected to 8 neighbours

change the probability to rewire to find the optimal value
(hill climb, easy to search)

Watts and Strogatz, Nature, 393, 440--442

Structured Node model (SN model)

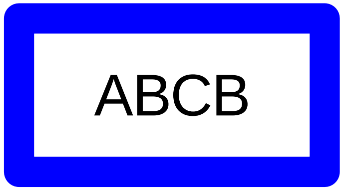


nodes have a structure

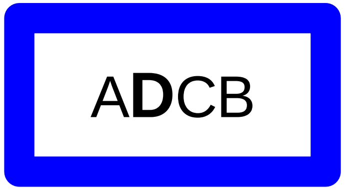
creation of new nodes and edges based on structure

Structured Node model (SN model)

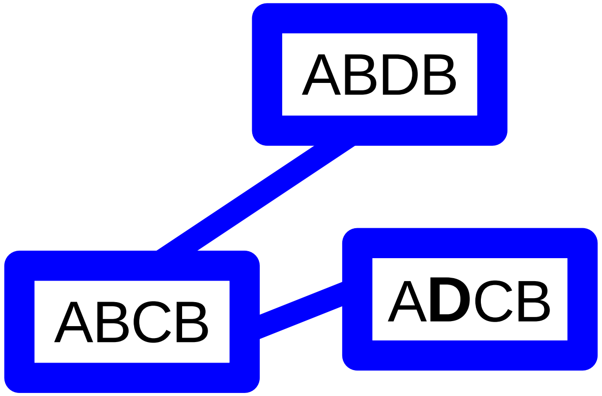
pick an existing node



mutate its structure to form a new node



add node to network, adding edges to other nodes based on a distance measure



repeat until you have the desired number of nodes

Structured Node model (SN model)

take two node structures

ABCB

ADCB

break them into codons

[AB],[CB]

[AD],[CB]

every where that the codons are not equal increase the hamming distance by one

[AB] =/= [AD]
[CB] == [CB]
Hamming distance = 1

if the hamming distance is below a threshold value then create an edge between the two nodes

Structured Node model (SN model)

hamming distance is the number of differences between strings of equal length

ADBCDABCDACDB
ADDCDBBADCCDB

if they are of unequal length, then we shorten the longer string

ADBCDABCDACDB
ADDCDBBADCCDBABCD

Structured Node model (SN model)

complex model

many parameters

found networks with values close to those of the targets

```
Counter = 38
Random_seed = 1,3
Network_size = INCREMENTAL
Initial_node = ABCDABCDABCD
Num_initial_nodes =
Prob_edge_duplication = 0.7
Final_remove_min_nodes = 0
Running_remove_min_nodes = 0
Running_remove_max_nodes = 0
Num_new_edges_for_each_new_node = 1
Num_runs_each_network = 1999
Frequency_save = 500
Type_mutation = RANDOM
Mutation_fix_number = 1
Prob_to_mutate = 0.2
Prob_to_add = 0.8
Prob_to_delete = 0
Prob_to_duplicate = 0
Alphabet = A,B,C,D
Chosen_node = RANDOM
Max_num_attempts = 1000
Type_distance = HAMMING
Direction = HAMMING
                         12,1          9%
```

Structured Node model (SN model)

creating networks from the SN model is more difficult

many parameters means multi-objective optimisation

used a genetic algorithm to find a suitable set of parameters

Global topological measurements

For each network model we created 10 networks.

We took measurements for each of the networks, and then averaged them.

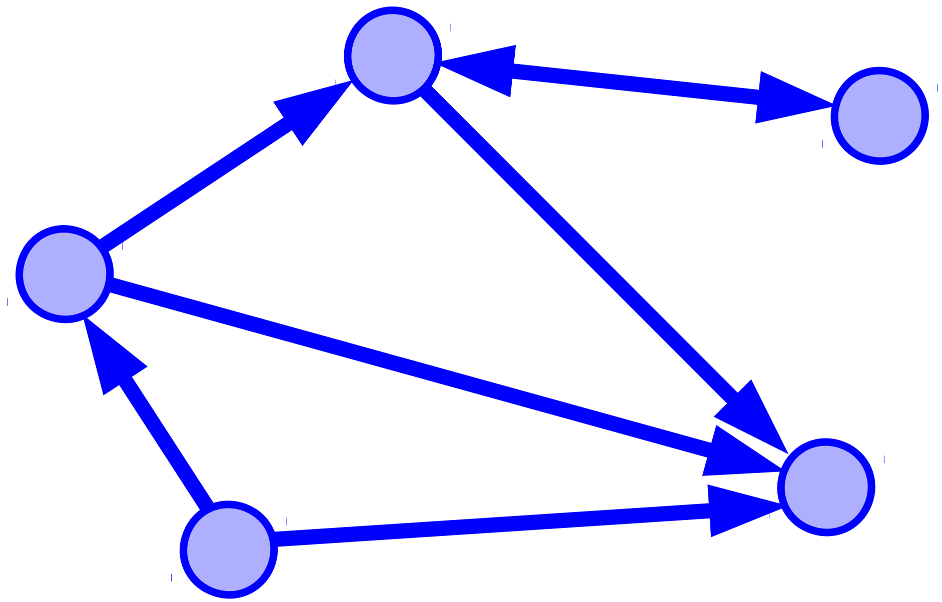
Global topological measurements

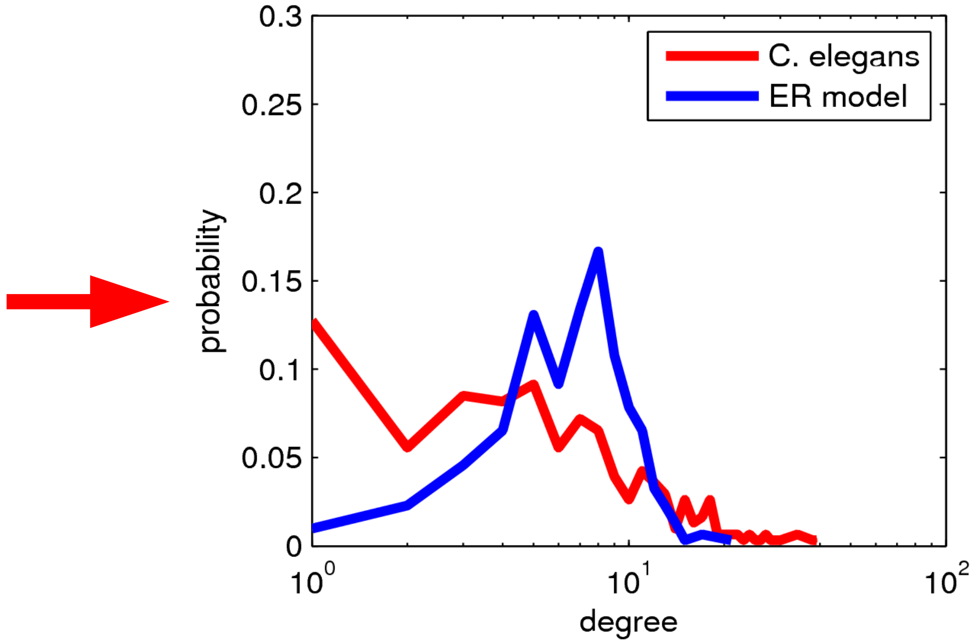
Network	Average Degree	Average Path Length	Average Clustering Coefficent
C. Elegans	7.66	2.46	0.284
Erdos-Renyi	7.56±0.15	2.41±0.02	0.05±0.00
Watts-Strogatz	8	2.78±0.01	0.29±0.01
Structured Nodes	6.43±0.41	3.73±0.12	0.36±0.03

Watts-Strogatz is a very good match to *C. elegans*

SN model is in second place

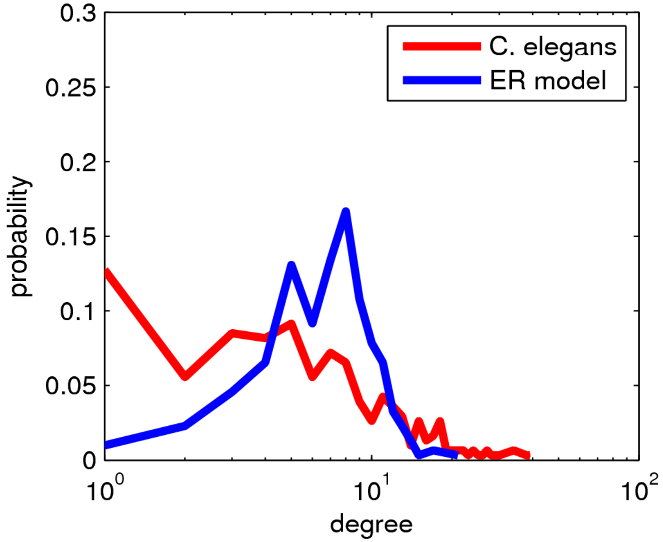
Distribution of topological measurements

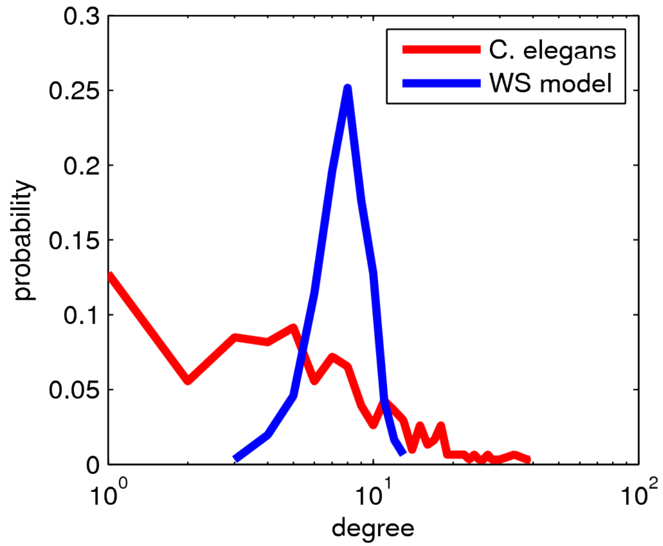


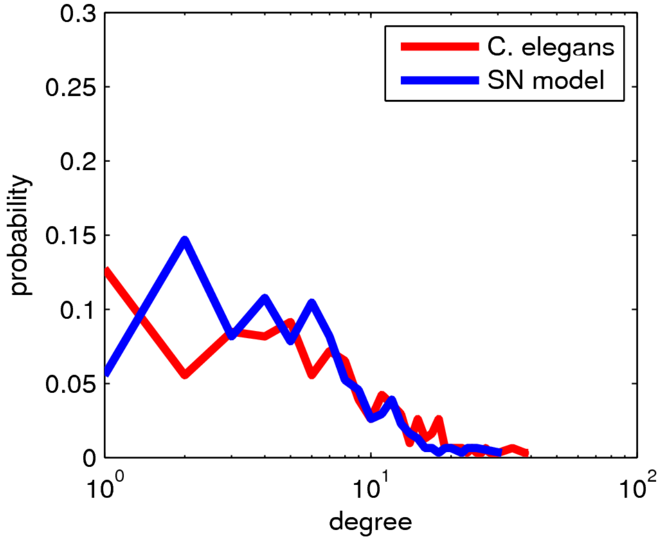


Tells us more than just the global averages, but are harder to analyse as they provide multidimensional data

Degree distribution



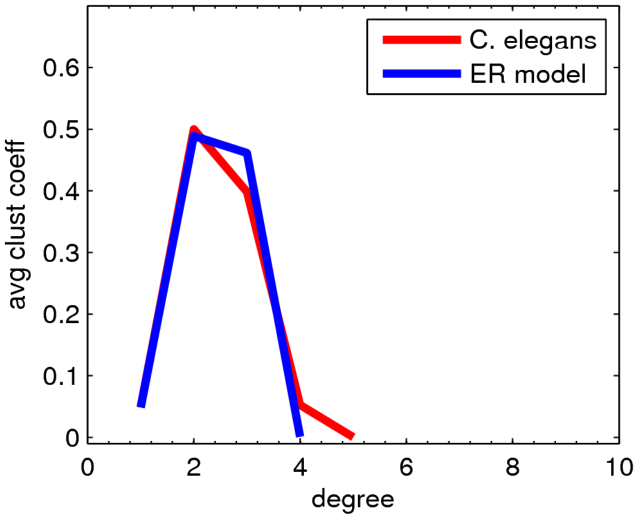


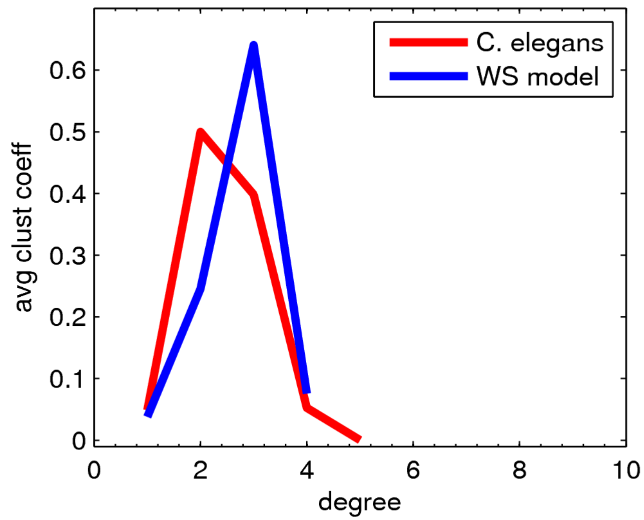


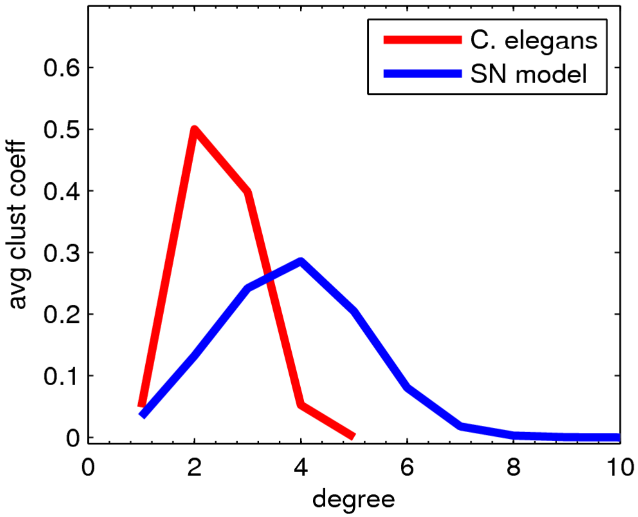
ER & WS models not like *C. elegans*

SN model similar to *C. elegans*

Path Length distribution



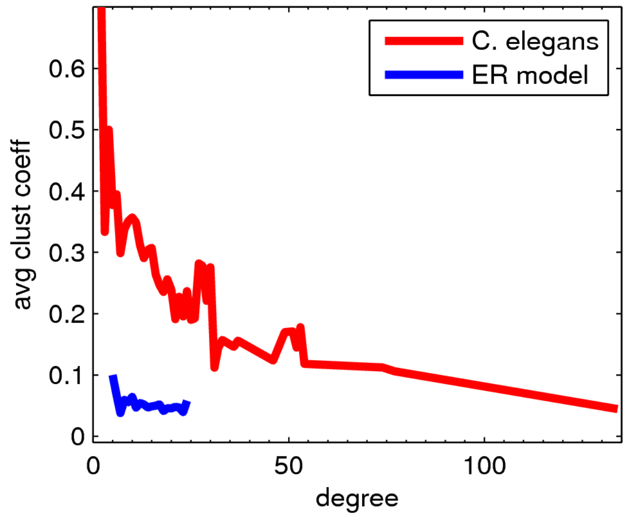


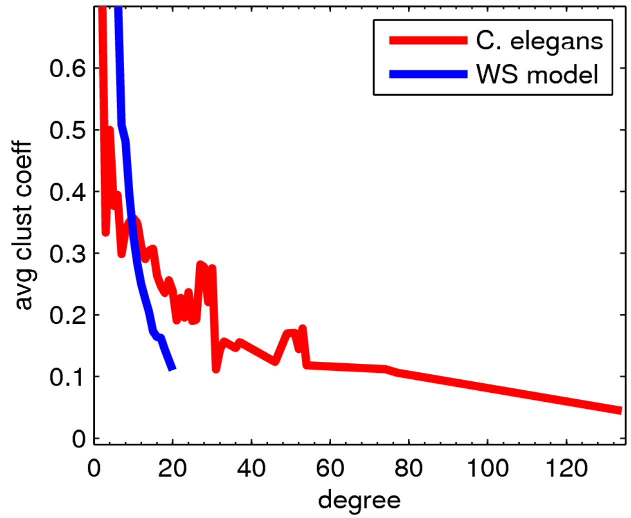


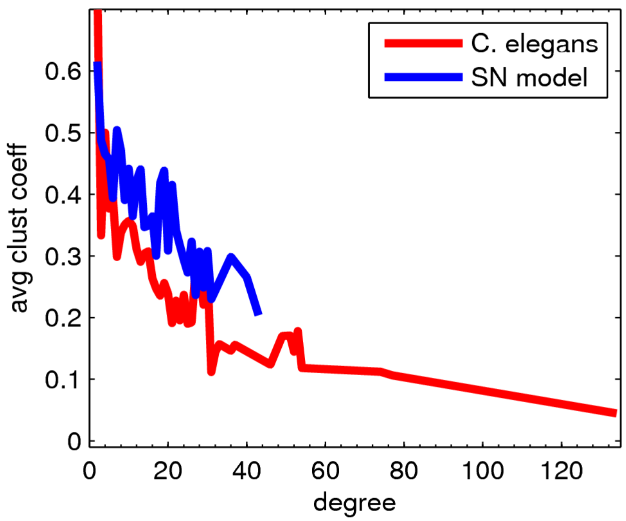
ER & WS model like *C. elegans*

SN model not like *C. elegans*

Cluster Coefficient distribution





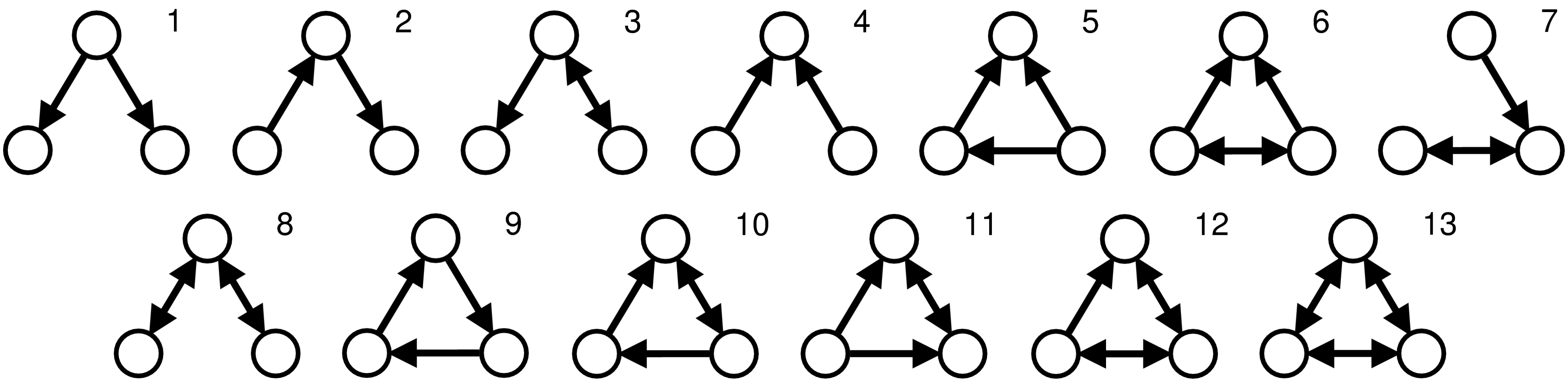


ER model not like *C. elegans*

WS model like *C. elegans*

SN model more like to *C. elegans*

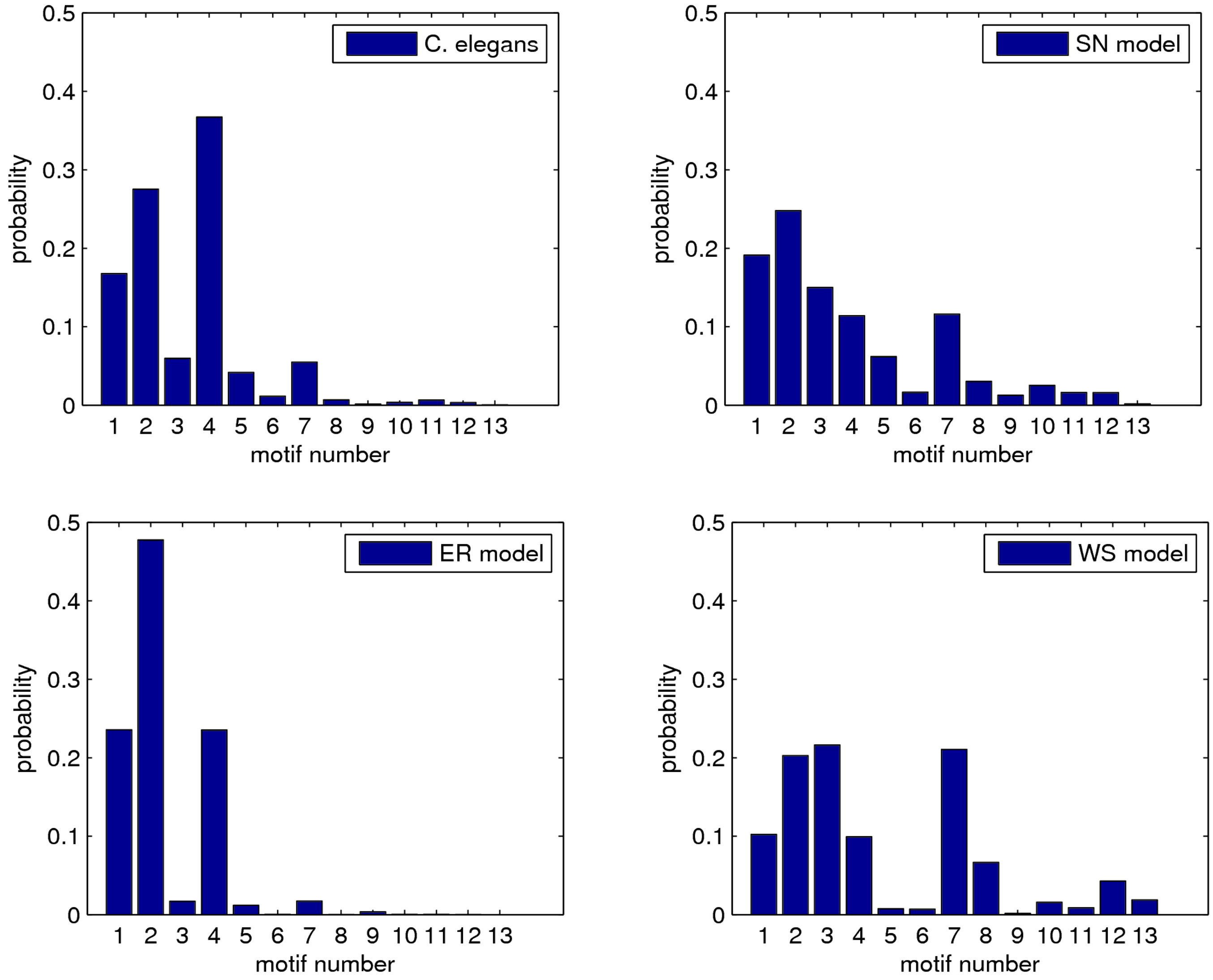
Motifs



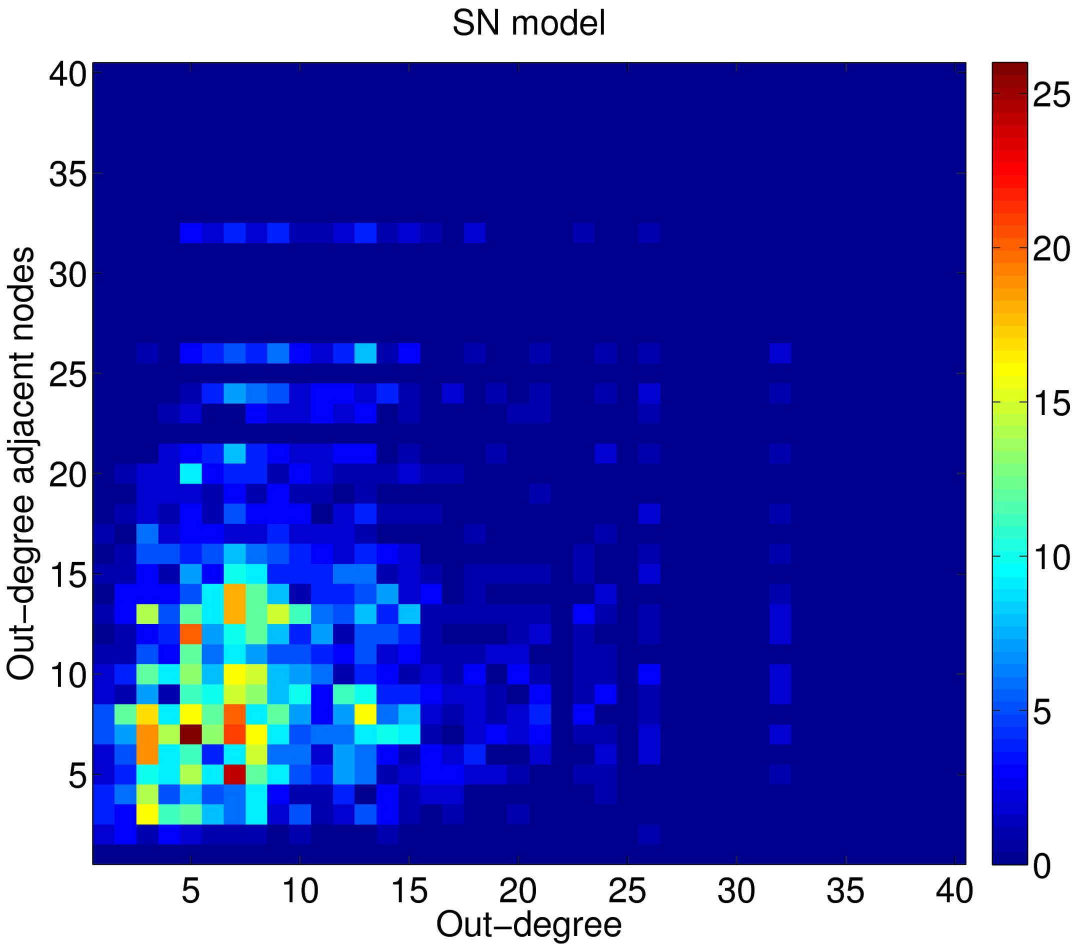
“patterns of interconnections that are found in significantly higher numbers in complex networks than random networks”

Milo, R. et al. Network Motifs: Simple Building Blocks of Complex Networks, Science, 2002, 298, 824-827

Motif distribution

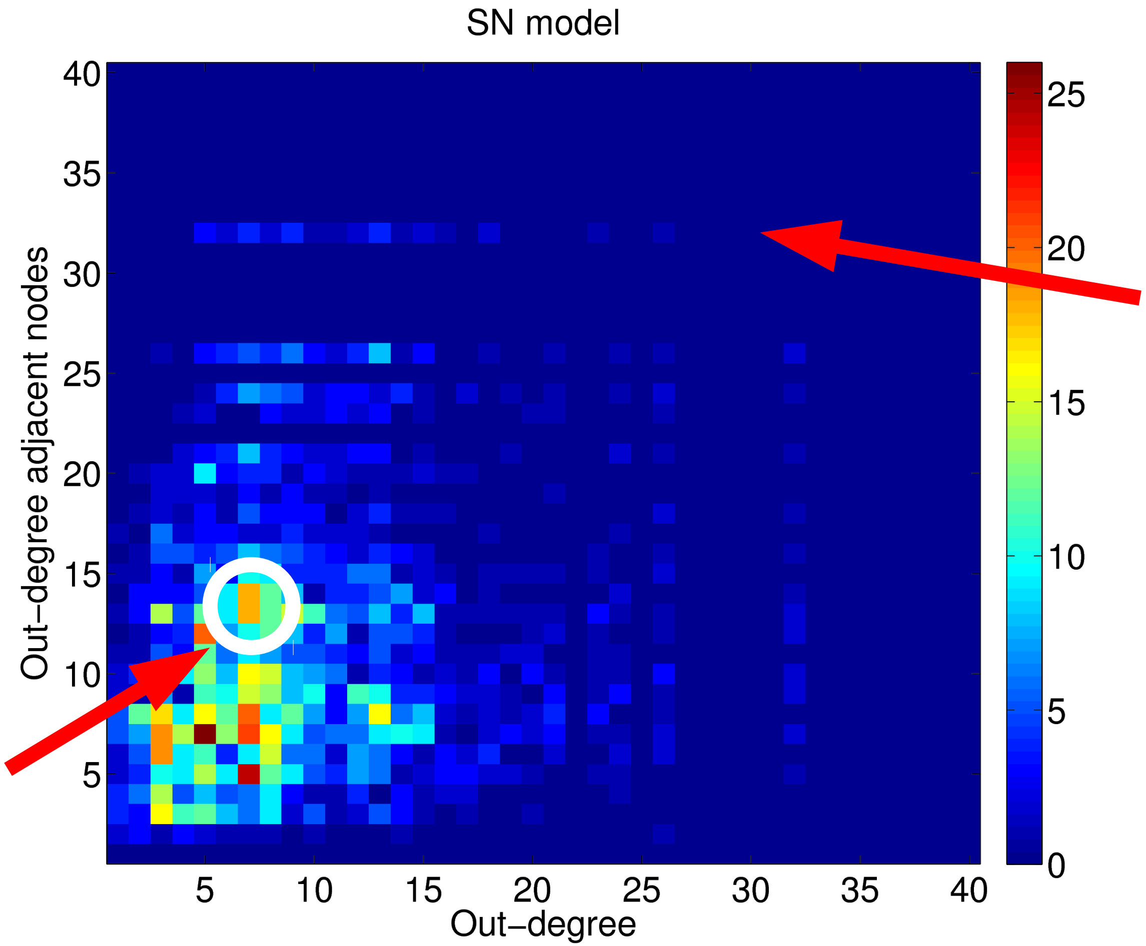


Outgoing edge heatmaps



shows how nodes are connected to other nodes based on their degree

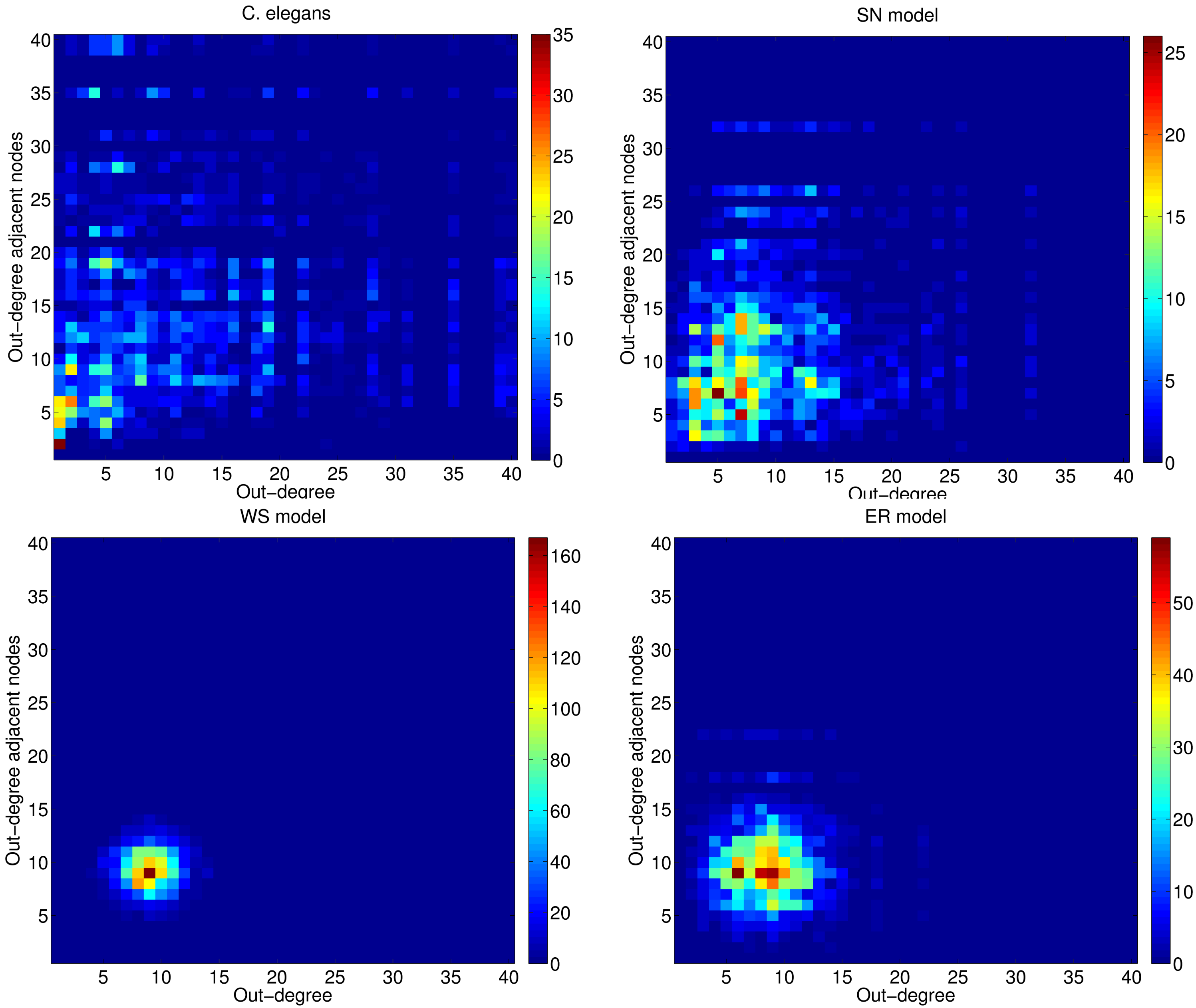
Outgoing edge heatmaps



There are many nodes of degree 7 connected to nodes of degree 13 & 14.

Nodes of degree 32 have very few incoming nodes, but those are all from nodes with smaller degrees.

Outgoing edge heatmaps



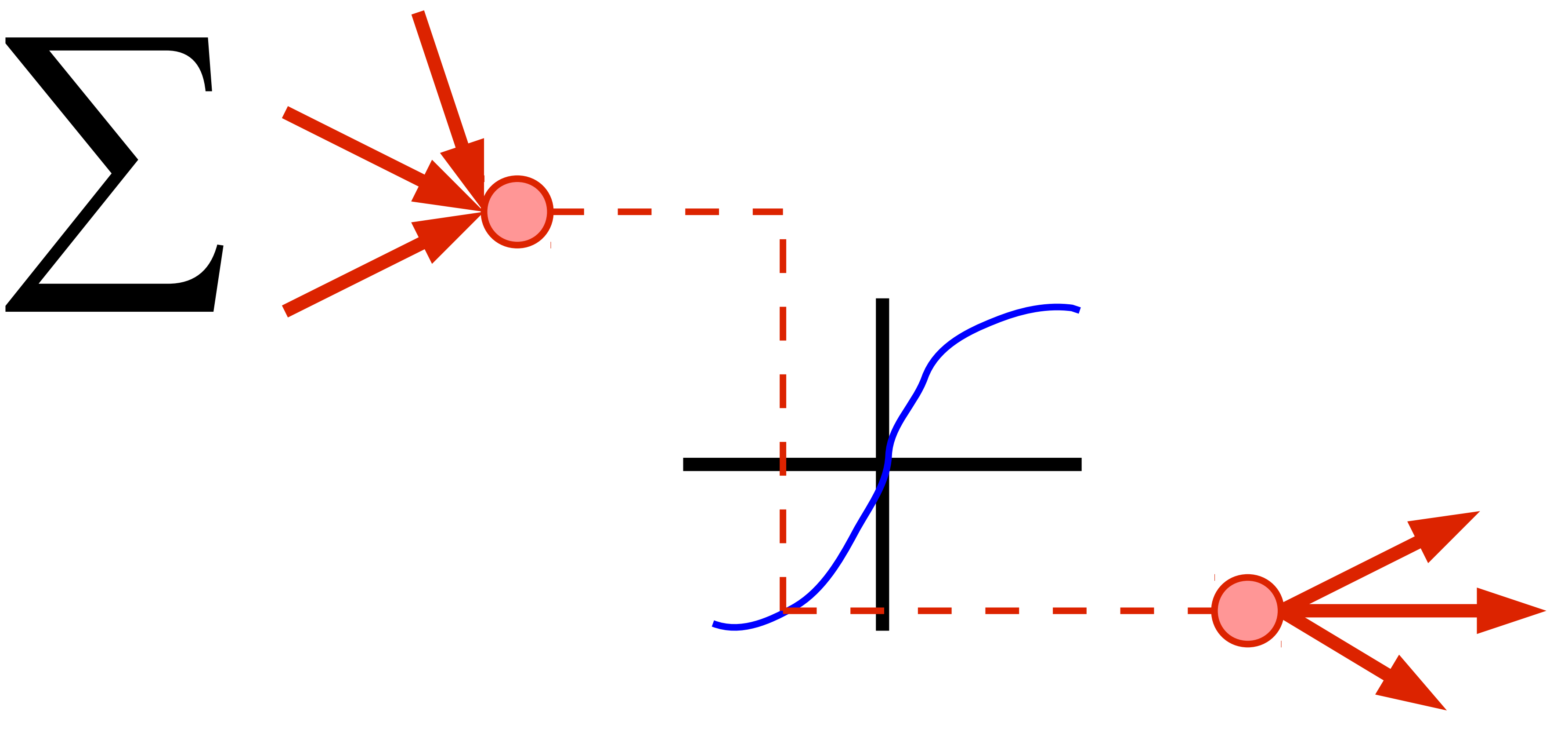
Distributions of measurements

network	avg. degree	avg. path length	avg. clustering coefficent	similar to degree distribution
C. Elegans	7.66	2.46	0.284	--
Erdos-Renyi	7.56±0.15	2.41±0.02	0.05±0.00	No
Watts-Strogatz	8	2.78±0.01	0.29±0.01	No
Structured Nodes	6.43±0.41	3.73±0.12	0.36±0.03	Yes

the SN model is the best fit of the distribution, depending on the measurements considered

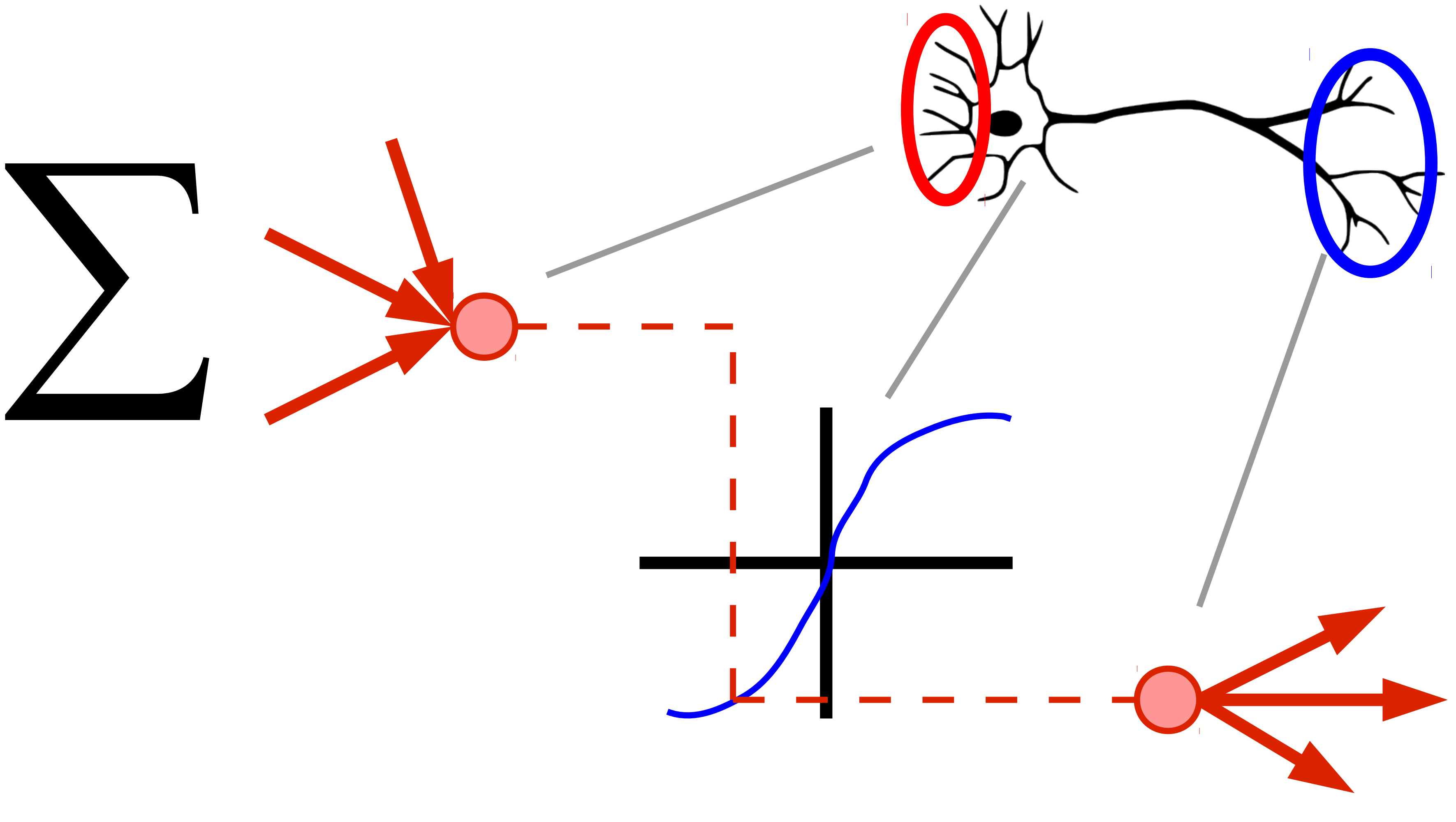
Random Recurrent Neural Networks

simple model of a neural network



Random Recurrent Neural Networks

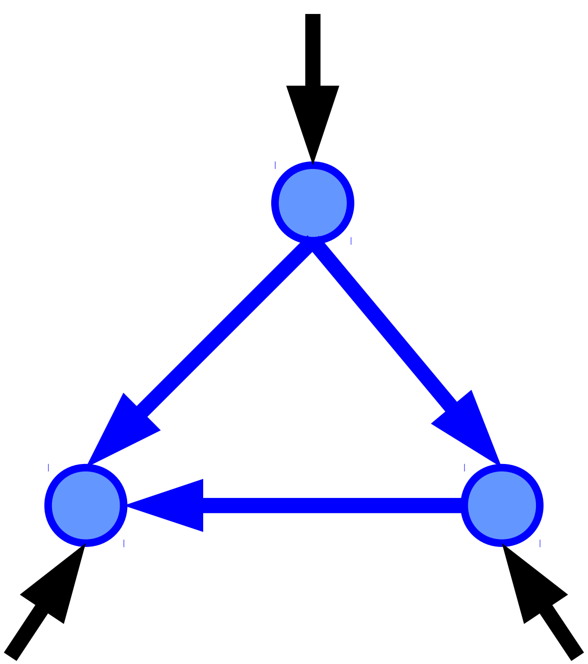
simple model of a neural network



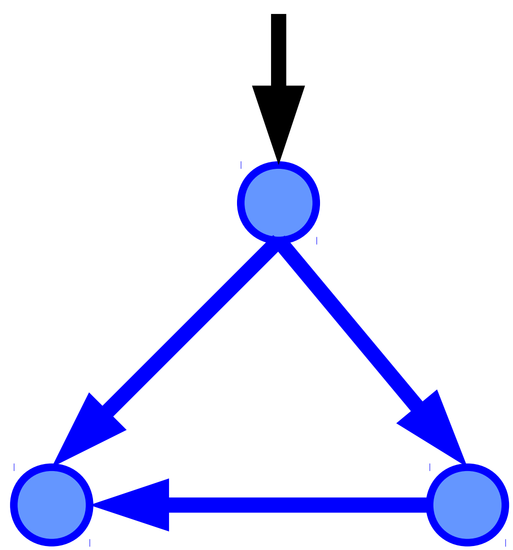
Adding an Influence

Three different methods of adding an influence were used

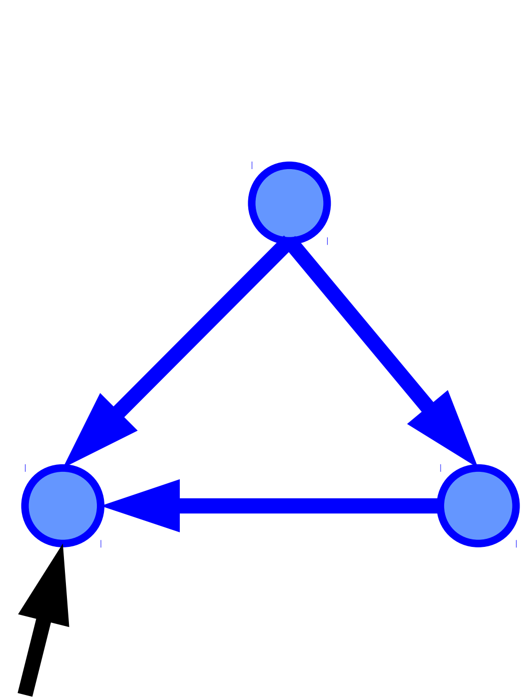
All Neurons



Most Outgoing Synapses

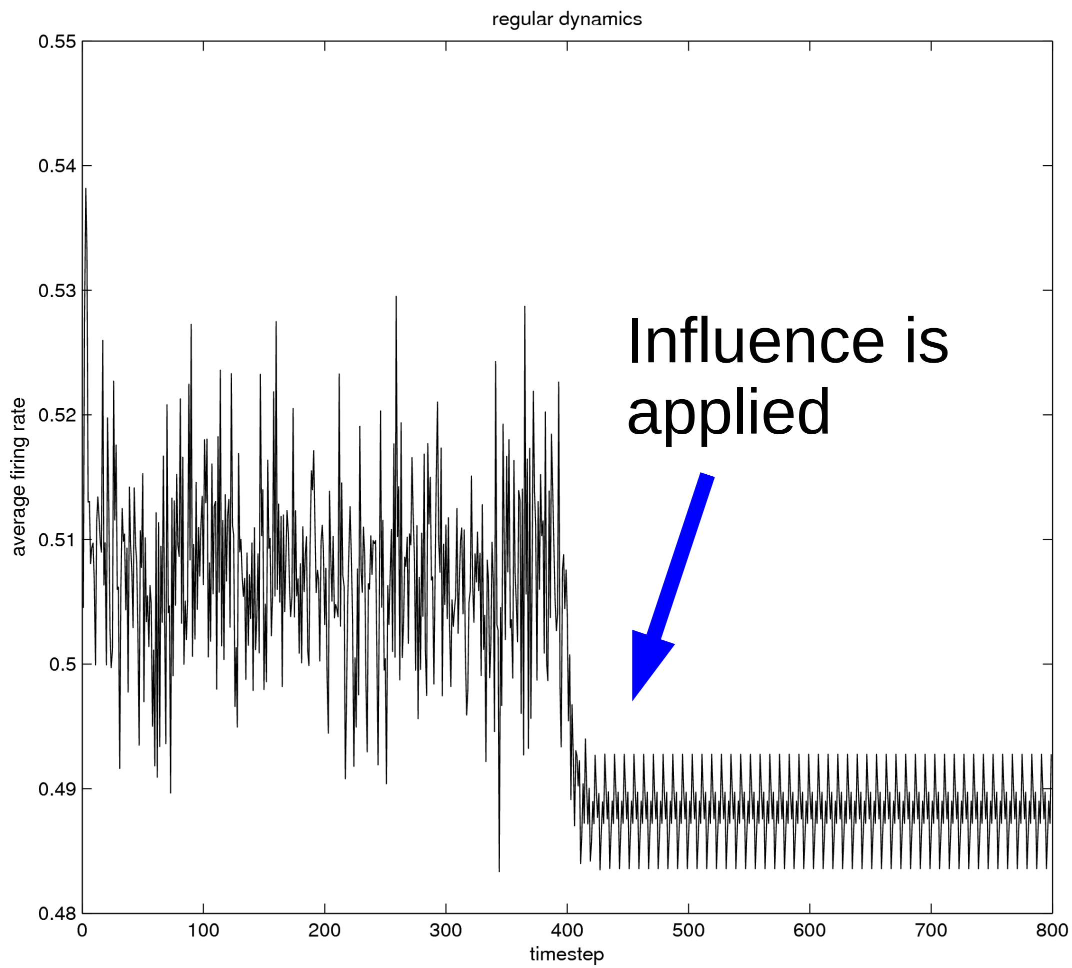


Least Outgoing Synapses



Observing the Dynamics

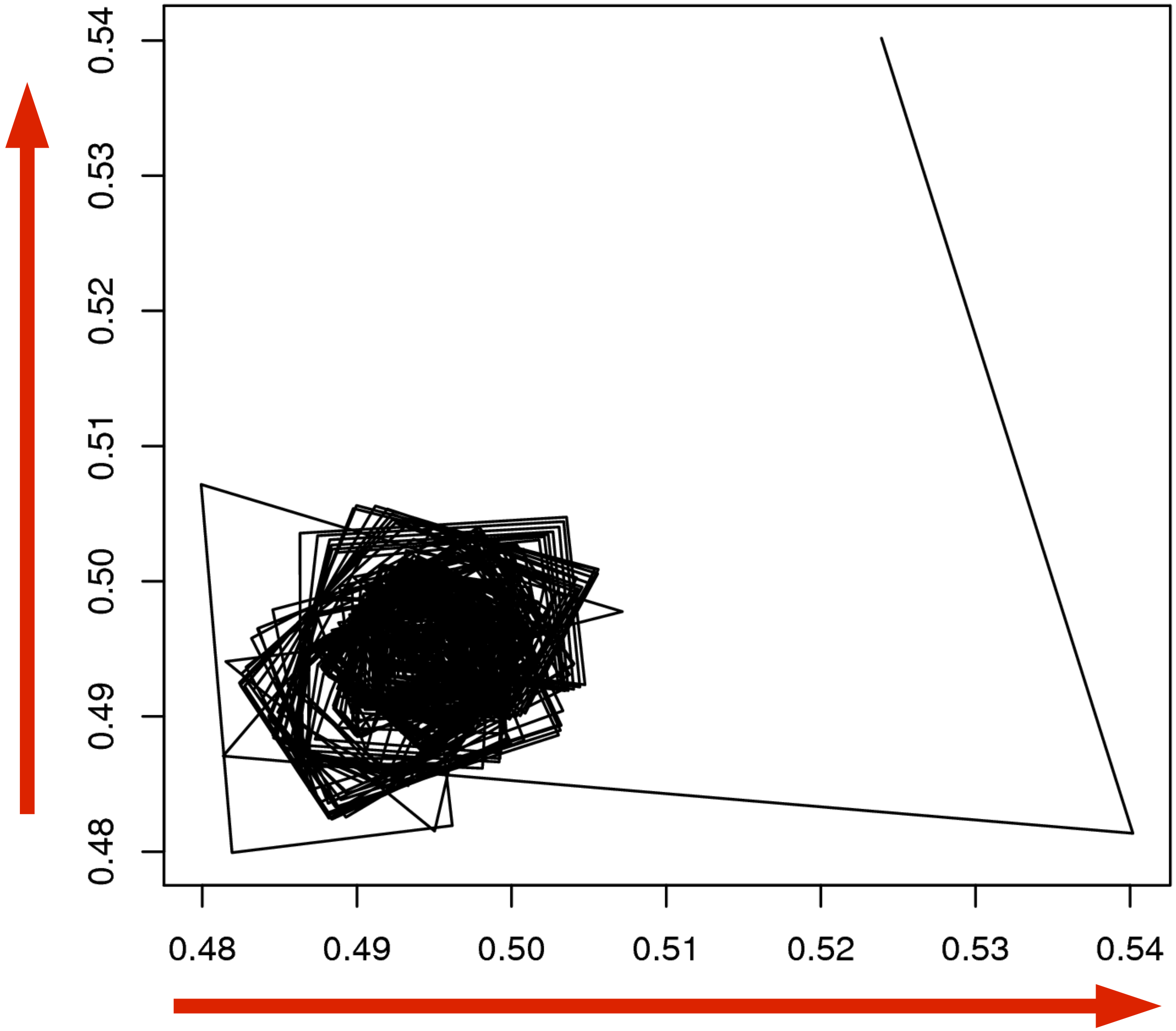
Average
firing
rate



Time

Trajectories of Dynamics

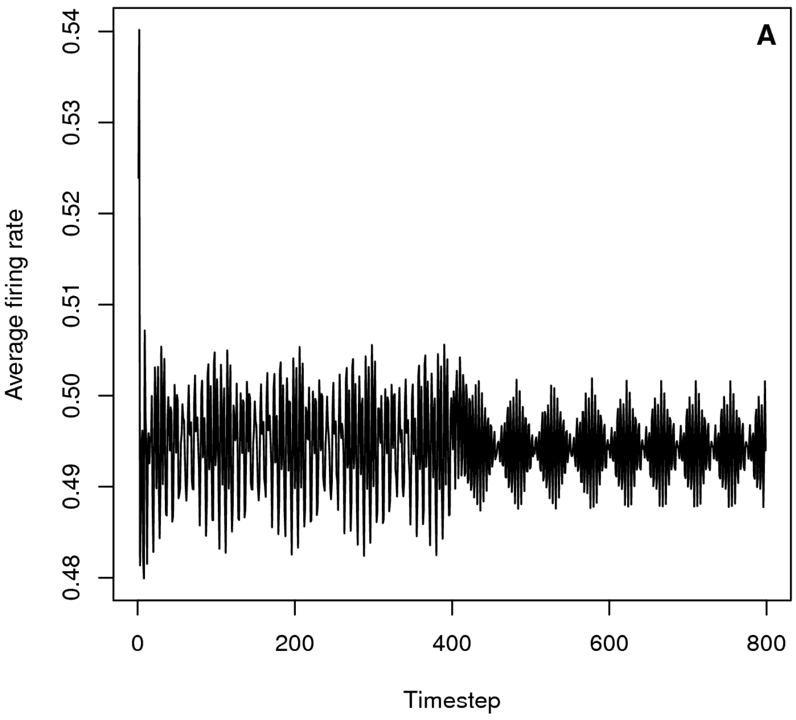
Average firing rate at time *t+1*



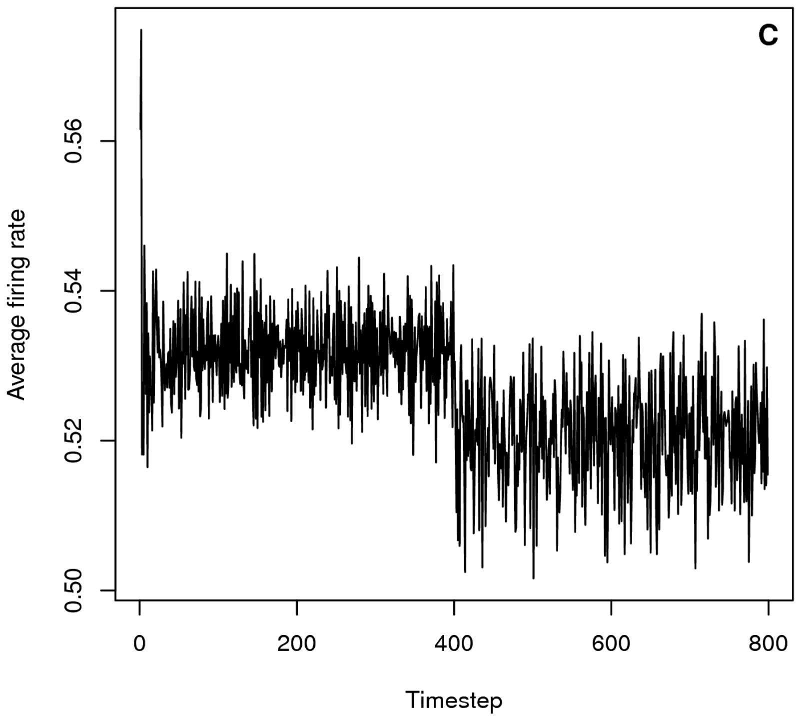
Average firing rate at time *t*

Regular Dynamics

Regular



Not Regular



Exploring the dynamics

network	% regular all nodes	% regular most connected nodes	% regular least connected nodes
C. Elegans	100	100	90
Erdos-Renyi	94	67	54
Watts-Strogatz	75	27	29
Structured Nodes	82	52	40

C. elegans has by far the most regular dynamics!

Conclusions

The WS model, though widely used, fails to model any distributions of measurements.

The SN model closely matches the distributions of measurements

None of the examined models come close to matching the regularity of the dynamics shown by the *C. elegans* network.

Future models may need to draw inspiration from neural development.

Thanks for listening,

any questions?

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