

# What is the dimension of citation space?

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Citation networks are constrained in time because documents can only cite older documents. In the same way the causal set approach to quantum gravity views space-time as a discrete set of elements with a particular causal structure. Both causal sets and citation networks form directed acyclic graphs. In the quantum gravity context the discrete structure is sufficient to fix the properties of the large scale continuous space-time we experience. In particular the space-time dimension can be estimated using just the causal relationships between the discrete points. We will show how to adapt these estimates of manifold dimension in order to characterise the structure of different citation networks.

We then apply these measures to three different types of citation network: academic papers on arXiv, US patents, and US Supreme Court judgements. We find that independent estimates of dimension converge on a consistent value for a given citation network. However, networks that otherwise appear very similar in structure turn out to have significantly different dimensions. We will show that by using network analysis methods that take the causal constraints into account, our approach reveals interesting distinctions in the structure of these temporal networks. This analysis differs from other conceptions of a network's 'dimension' by recognising the causal constraints on directed acyclic graphs and incorporating time explicitly as one of the dimensions we measure.

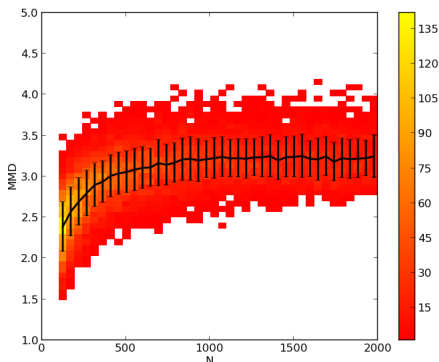


Figure 1: The Myrheim-Meyer dimension for the arXiv hep-ph citation network tends towards 3 for large intervals (subsections of the network).

Network	Dimension
arXiv hep-th	2
arXiv hep-ph	3
US Patents	5
US Supreme Court judgements	2

Table 1: Although subsections of a given citation network have similar dimensions, different citation networks often have very different dimensions even though they may appear similar according to other measures.