Simmelian brokerage and social capital: Reconciling social cohesion and structural holes

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Social capital

- Investments in social relations yield expected returns in the marketplace (Lin, 2001; Lin et al., 2001).
- Social capital "inheres in the structure of relations between actors and among actors, [and] like other forms of capital, [it] is productive, making possible the achievement of certain ends that in its absence would not be possible" (Coleman, 1998, p.S98).
- Social scientists have long agreed on the salience of social structure as a source of social capital (Granovetter, 1973, 2005; Lin, 2001; Long Lingo & O'Mahony, 2010):
  - it facilitates or hinders the flow of information (Hansen, 1999; Reagans & McEvily, 2003; Tortoriello et al., 2012; Uzzi & Spiro, 2005);
  - it is a source of reward and punishment due to internalisation and enforcement of social norm (Gould, 1991; Ingram & Roberts, 2000);
  - it nurtures actors' trust, reputation, social credentials, status, identity and recognition through processes of third-party referrals and reinforcement of interactions (Lin, 2001; Lin et al., 2001; Uzzi, 1997).

The controversy

- Despite convergence on the salience of social structure, there is still controversy over which type of social structure matters as a source of social capital (Aral & Van Alstyne, 2011; Baum et al., 2012; Burt, 2005; Gargiulo & Benassi, 2000; Reagans & Zuckerman, 2001).
- “Verbindung zu dreien” (Simmel, 1923): expansion of a dyadic relationship into a three-party relationship.
- “The appearance of the third party indicates transition, conciliation, and abandonment of absolute contrast (although, on occasion, it introduces contrast).” (Simmel, 1923, p.145).
- Two functional roles played by the third party:
  - “non-partisan” or mediator with the *tertius iungens* (or “the third who joins”) orientation (Obstfeld, 2005);
  - broker with the *tertius gaudens* (or “the third who enjoys”) orientation (Burt, 1992).

Closed structures and social cohesion

- Actors separated by one intermediary are more likely to become connected with each other than actors that do not share any common acquaintance (Davis, 1970; Davis et al., 1971; Holland & Leinhardt, 1970, 1971; Luce & Perry, 1949; Watts, 1999; Watts & Strogatz, 1998).
- Closure-based sources of social capital:
  - normative control and deviance avoidance (Burt, 2005; Granovetter, 2005; Lin, 2001; Lin et al., 2001);
  - sense of belonging (Coleman, 1988) and trust (Burt and Knez, 1995; Coleman, 1990; Reagans & McEvily, 2003; Uzzi, 1997);
  - exchange of fine-grained, complex, tacit, and proprietary information (Hansen, 1999; Uzzi, 1997);
  - common culture and shared identity (Nahapiet & Ghoshal, 1998); and
  - cooperation (Coleman, 1988; Ingram & Roberts, 2000).

Open structures and brokerage

- Costs of closure:
  1. local redundancy: the more an actor's contacts are connected with each other, the less likely they are to take the actor closer to diverse sources of knowledge the actor is not already able to access (Burt, 1992; Granovetter, 1973).
  2. social pressure: a cohesive structure favours convergent thinking, group consensus, maintenance of the status quo rather than the exploration of novel paths leading to divergent solutions (Fleming et al., 2007; Sosa, 2011).
- There are benefits actors can extract from participating in open structures that are rich in cleavages and opportunities of brokerage (Burt, 1992, 2005, 2010; Long Lingo & O'Mahony, 2010; Stovel & Shaw, 2012).
- While the non-partisan *tertius iungens* aims "to save the group unity from the danger of splitting up" (Simmel, 1923, p.154), the *tertius gaudens* wishes to create or intensify discontinuities in the social structure by forging or preserving unique ties to disconnected others.
- Structural hole: "separation between non-redundant contacts", "a relationship of non-redundancy between two contacts", "a buffer" that enables the two contacts to "provide network benefits that are in some degree additive rather than overlapping" (Burt, 1992, p.18).

Hole-based sources of social capital

- Information benefits:
  - In open structures, connections tend to be weak (Granovetter, 1973) and are likely to link people with different ideas/interests/ perspectives (Burt, 2004).
  - Due to exposure to a greater variance and novelty of information, actors in brokered structures will be creative and successful (Burt, 2004; Fleming et al., 2007; Sosa, 2011).

- Control benefits:
  - third party's ability to gain an advantage by negotiating his or her relationships with disconnected others and turning their "forces combined against him into action against one another." (Simmel, 1923, p. 162).
  - extract social capital buried in the hole, by playing the disconnected parties' demands and preferences against one another (Burt, 1992).

Trade-off between closed and open structures

- Reconcile the two positions on social capital, and provide an integrative account of social cohesion and brokerage (Aral & Van Alstyne, 2011; Fleming et al., 2007; Perry-Smith, 2006; Rodan & Galunic, 2004; Tortoriello & Krackhardt, 2010; Vedres & Stark, 2010).

- Benefits originating from social structure are contingent on a number of social/structural/environmental conditions (Aral & Van Alstyne, 2011; Fleming et al., 2007; Perry-Smith, 2006; Rodan & Galunic, 2004).

- A suitable combination of the two types of structure can outperform each individual type in isolation (Reagans & Zuckerman, 2001; Tortoriello and Krackhardt, 2010; Vedres & Stark, 2010).

Our contribution to the debate

1. We formalise the trade-off between closed and open structures by proposing a functional relation between the measures with which these two types of structure have traditionally been operationalised.

2. We offer a new measure - Simmelian brokerage - for detecting the degree to which an individual's structural position lies at the interface between a closed and an open structure:

   – defined at the node level, it detects directly, based on the node's local neighbourhood, the extent to which the node belongs to multiple groups that are both tightly knit and disconnected from each other (Simmel, 1922).

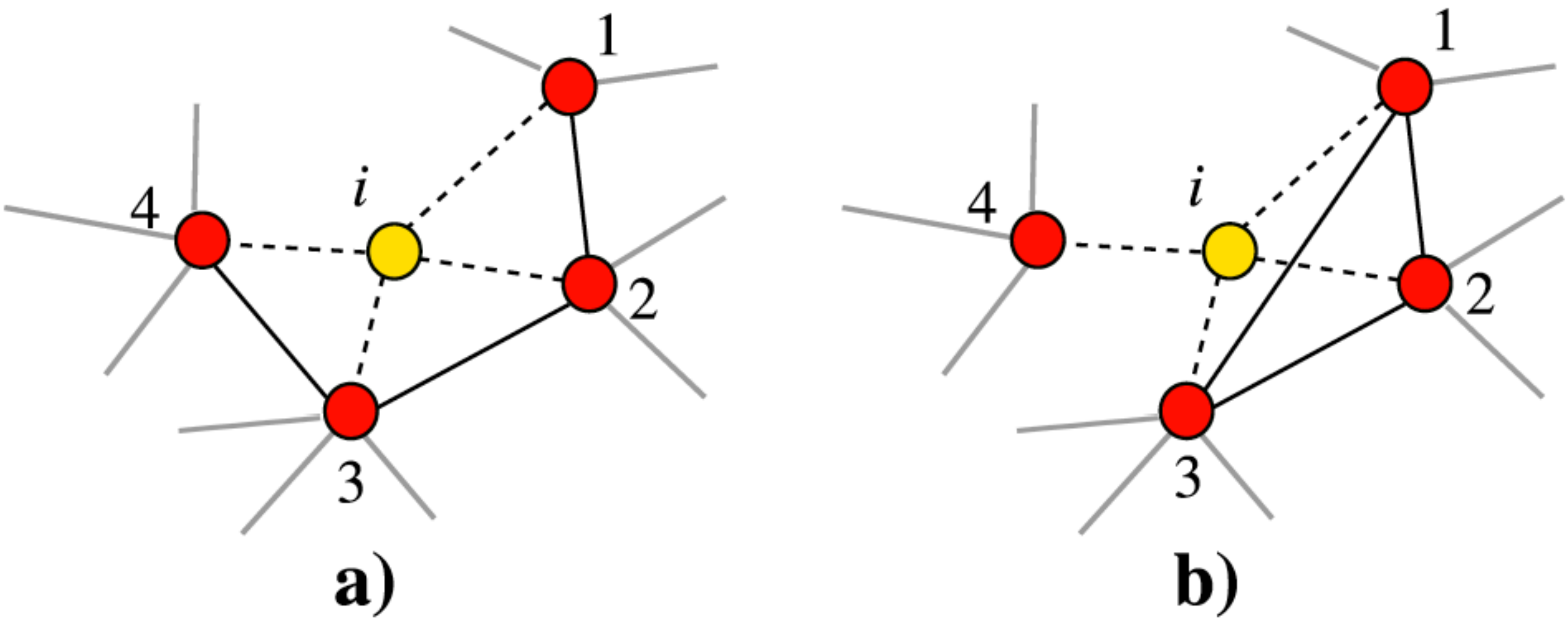
V. Latora, V. Nicosia, and P. Panzarasa (2013). Social cohesion, structural holes, and a tale of two measures, *Journal of Statistical Physics*, 151(3):745-764.

Measuring social cohesion: Clustering coefficient

$$C_i = \begin{cases} \dfrac{K[G_i]}{k_i(k_i-1)/2} & \text{for } k_i \geq 2 \\ 0 & \text{for } k_i = 0,1 \end{cases}$$

where $K[G_i]$ is the number of links in the unweighted, undirected subgraph G_i.

- Probability that two neighbours of node *i* are connected by a link, properly normalised by definition such that $0 \leq C_i \leq 1$ (Watts and Strogatz, 1998).
- Proportion of triads centred on *i* that close into triangles.



C_i only depends on the number of links in G_i, and not on which pairs of nodes are actually connected through such links in G_i.

$C_i = 1/2$ in both a) and b).

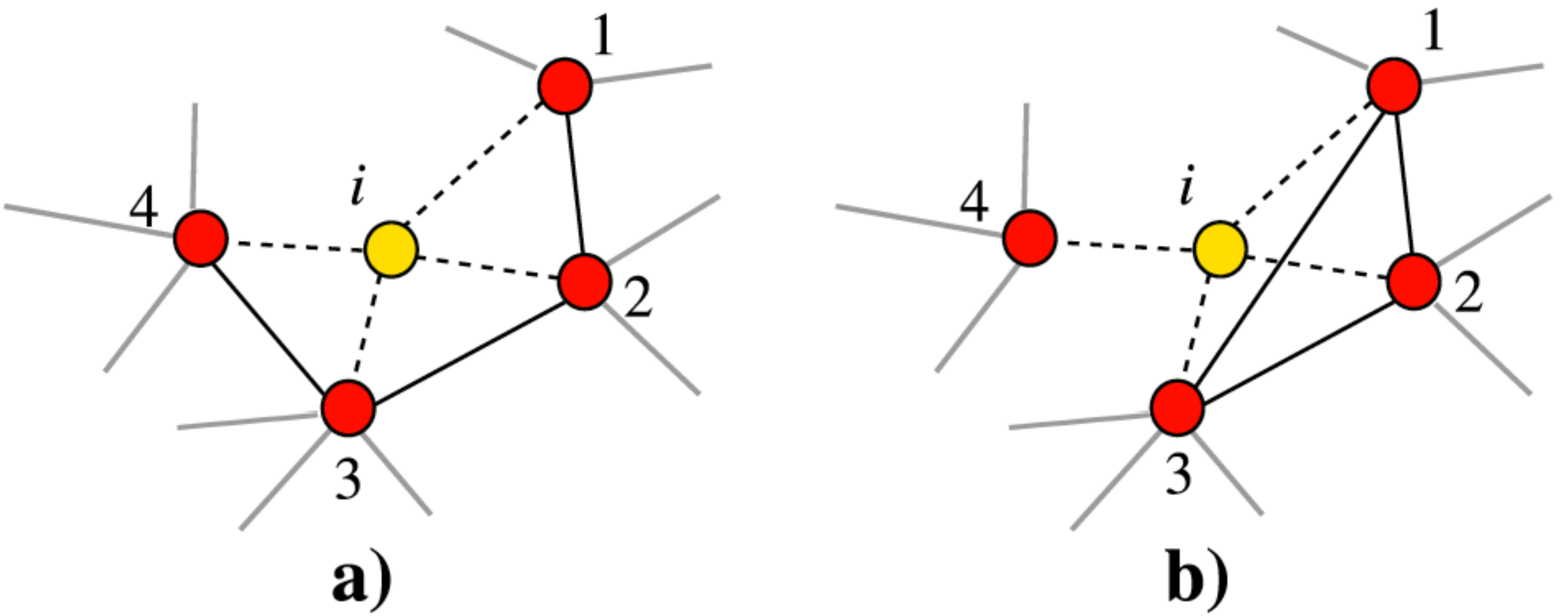
Measuring social cohesion: Local efficiency

- Local efficiency of node *i*: Efficiency of unweighted, undirected subgraph G_i , i.e., the average of the inverse of the distances between the nodes of G_i (Latora & Marchiori, 2001, 2003):

$$E_i = E[G_i] = \frac{1}{k_i(k_i-1)} \sum_{\ell \in \mathcal{N}_i} \sum_{\substack{m \in \mathcal{N}_i \\ m \neq \ell}} \varepsilon_{\ell m} = \frac{1}{k_i(k_i-1)} \sum_{\ell \in \mathcal{N}_i} \sum_{\substack{m \in \mathcal{N}_i \\ m \neq \ell}} \frac{1}{d_{\ell m}}$$

- where ε_{lm} is the reachability between nodes *l* and *m*, and is equal to the inverse of the distance d_{lm} between the two nodes.

- Local efficiency takes values in the same range as the clustering coefficient: $0 \leq E_i \leq 1$.



- $E_a^i = 13/18$; $E_b^i = 1/2$
- Yet, in both a) and b) $C_i = 1/2$.
- Like clustering, for a fixed number of nodes, efficiency becomes larger as the number of links increases.
- Unlike clustering, for a fixed number of nodes and links, efficiency depends on where the links are actually located.

Measuring structural holes: Effective size

For a directed weighted graph, the effective size of node *i*'s network indicates the extent to which each of the first neighbours of *i* is redundant with respect to the other neighbours (Burt, 1992):

$$\mathcal{S}_i = \sum_{j \in \mathcal{N}_i} \left[1 - \sum_{\ell} p_{i\ell} m_{j\ell} \right] \qquad l \neq i, j$$

where p_{il} is the entry of the transition matrix P, and measures the proportion of *i*'s time and energy invested in relationship with node *l*:

$$p_{i\ell} = \frac{w_{i\ell} + w_{\ell i}}{\sum_m (w_{im} + w_{mi})} \qquad i \neq m$$

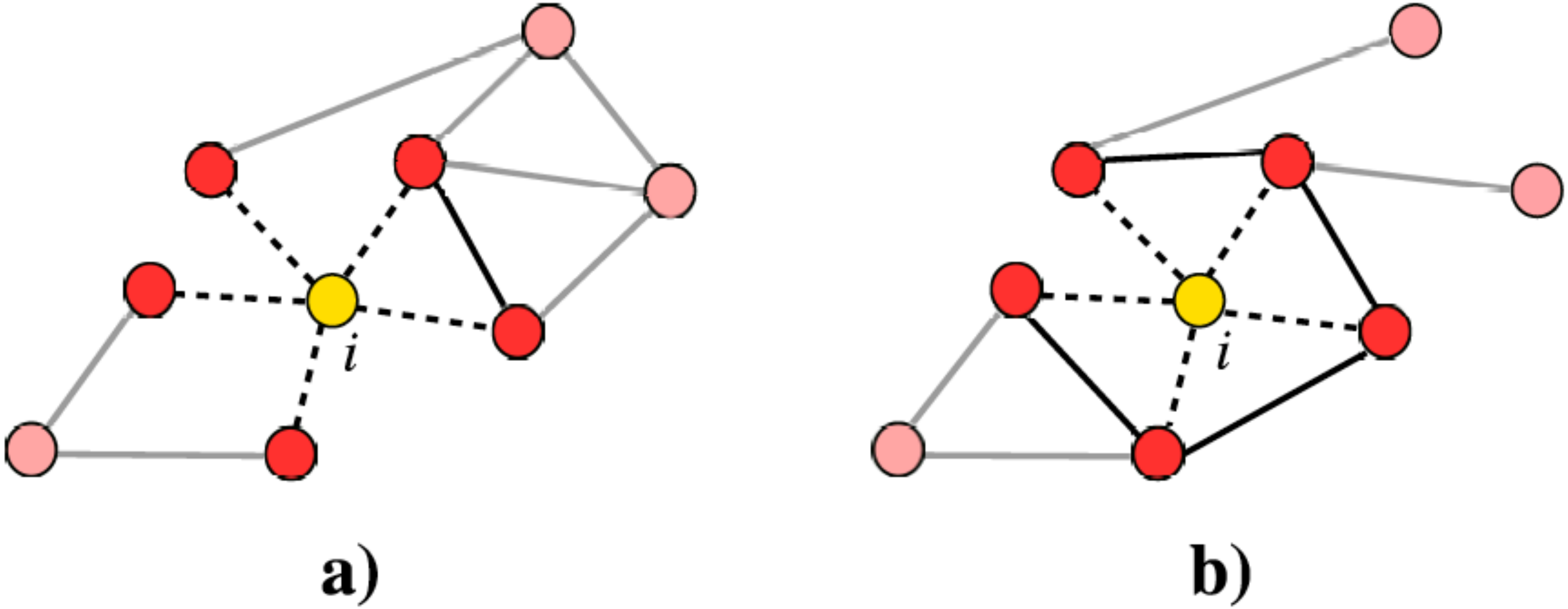
and m_{jl} is the entry of the marginal strength matrix M:

$$m_{j\ell} = \frac{w_{j\ell} + w_{\ell j}}{\max_m (w_{jm} + w_{mj})} \qquad j \neq m$$

If *i* is an isolate, $S_i \equiv 0$ by definition. Otherwise, $1 \leq S_i \leq k_i \ \forall i$.

For undirected graphs ($w_{il} = w_{li}$), $p_{il} = w_{il} / s_i^{out}$, and $m_{jl} = w_{jl} / max_m w_{jm}$.

For undirected and unweighted graphs ($w_{il} = a_{il}$), $p_{il} = a_{il} / k_i$, *and* $m_{jl} = a_{jl}$ (for any node *j* that is not an isolate).



$S_i^a = 1+1+1+(1-1/5)+(1-1/5) = 3+8/5 = 23/5$

$S_i^b = (1-1/5)+(1-2/5)+(1-2/5)+(1-2/5)+(1-1/5) = 17/5.$

Clustering and effective size in unweighted graphs

- Both C_i and S_i depend on number of triangles containing node *i*: The more triangles, the larger C_i and the smaller S_i.

- Inverse relation between C_i and S_i: the larger the clustering coefficient of a node, the smaller the effective size of the node's network.

A formal relation

- The local clustering coefficient of node I can be expressed as:

$$C_i = \begin{cases} \frac{\sum_{j,\ell} a_{ij} a_{j\ell} a_{\ell i}}{k_i(k_i - 1)} & \text{for } k_i \geq 2 \\ 0 & \text{for } k_i = 0, 1 \end{cases}$$

where $\sum_{j,\ell} a_{ij} a_{j\ell} a_{\ell i}$ denotes the number of closed walks of length 3 from node *i* to itself (i.e., twice the number of triangles containing node *i*).

- For undirected and unweighted graphs, we have:

$$\mathscr{S}_i = \sum_j a_{ij} \left[1 - \sum_\ell p_{i\ell} m_{j\ell} \right] = k_i - \sum_j a_{ij} \sum_\ell p_{i\ell} m_{j\ell} =$$

$$= k_i - \sum_j \sum_\ell a_{ij} \frac{a_{i\ell}}{k_i} a_{j\ell} = k_i - \frac{1}{k_i} \sum_j \sum_\ell a_{ij} a_{j\ell} a_{\ell i}$$

$$= k_i - (k_i - 1) C_i$$

- When S_i is normalised in [0,1], we have: $\mathscr{S}'_i = 1 - \frac{k_i - 1}{k_i} C_i \simeq 1 - C_i$ (for large k_i).

Reconciling social cohesion and structural holes

- Capture brokerage opportunities among otherwise disconnected socially cohesive groups of nodes.

- New measure for brokerage that:
  - like clustering and effective size, will be sensitive to structural gaps in a node’s local network; but
  - unlike clustering and effective size, will also be sensitive to variations in the position of links across local networks of the same density.

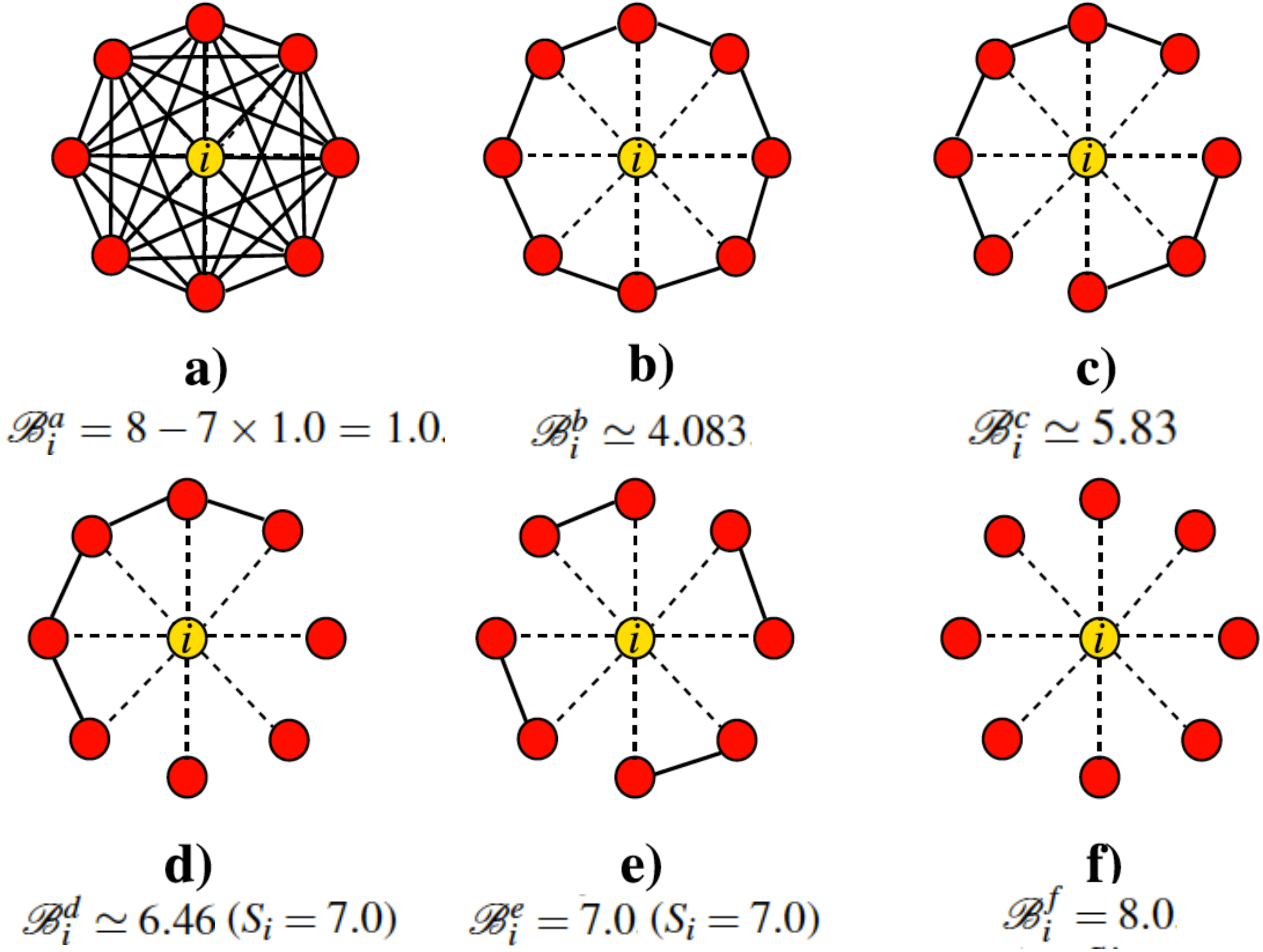
Simmelian brokerage

- The higher the local efficiency of a node, the fewer the opportunities a node has to act as a broker, and vice versa.

- Based on $\mathscr{S}_i = k_i - (k_i - 1)C_i$, we define Simmelian brokerage as:

$$\mathscr{B}_i = k_i - (k_i - 1)E_i$$

- B_i is sensitive to the extent to which a node acts as a broker between Simmelian ties (Krackhardt, 1998, 1999) or, alternatively, between otherwise disconnected groups of densely connected nodes.



a) $\mathscr{B}_i^a = 8 - 7 \times 1.0 = 1.0$

b) $\mathscr{B}_i^b \simeq 4.083$

c) $\mathscr{B}_i^c \simeq 5.83$

d) $\mathscr{B}_i^d \simeq 6.46$ $(S_i = 7.0)$

e) $\mathscr{B}_i^e = 7.0$ $(S_i = 7.0)$

f) $\mathscr{B}_i^f = 8.0$

Effective strength and weighted clustering

- Effective strength: $$\mathscr{S}_i^w = \sum_j w_{ij} \left[1 - \sum_\ell p_{i\ell} m_{j\ell} \right] = s_i - \sum_j \sum_\ell w_{ij} p_{i\ell} m_{j\ell}$$
- Weighted clustering coefficient (Opsahl & Panzarasa, 2009; Saramäki et al., 2007):

$$C_i^w = C_i^B = \frac{1}{s_i(k_i-1)} \sum_{j,\ell} \frac{w_{ij}+w_{i\ell}}{2} a_{ij} a_{j\ell} a_{\ell i}$$ (Barrat et al., 2004)

$$C_i^w = C_i^O = \frac{2\sum_{j,\ell}(w_{ij} w_{j\ell} w_{\ell i})^{1/3}}{k_i(k_i-1)}$$ (Onnela et al, 2005)

$$C_i^w = C_i^Z = \frac{\sum_{j\neq i}\sum_{j\neq \ell, \ell\neq i}(w_{ij} w_{j\ell} w_{\ell i})}{(\sum_{j\neq i} w_{ij})^2 - \sum_{j\neq i} w_{ij}^2}$$ (Zhang and Horvath, 2005)

$$C_i^w = C_i^H = \frac{\sum_{j,\ell} w_{ij} w_{j\ell} w_{\ell i}}{\max(w) \sum_{j,\ell} w_{ij} w_{\ell i}}$$ (Holme et al., 2007)

- Relation between weighted strength and clustering: $$\mathscr{S}_i^w = F(k_i, s_i, C_i^w)$$
- Simmelian brokerage: $$\mathscr{B}_i^w = F(k_i, s_i, E_i^w)$$

where E_i^w is the local efficiency of node *i* in the weighted graph, which is based on weighted distances d^w_{jl} instead of topological distances d_{jl}.

Summary

- Emphasis not only on ties, but also on structural cleavages.
- The study of formal relations between graph measures can help unveil the intimate connections between sociological concepts and lead to the development of new concepts and measures.
- From formalisation of the relation between closed and open structures to the proposal of a new measure for topological configurations at the interface between the two types of structure.
- Simmelian brokerage:
  - Identify brokerage positions in which a node can intermediate between otherwise disconnected cohesive groups of contacts.
  - Differentiate between brokerage positions of nodes with the same degree and clustering coefficient, but with a different configuration of links in their local neighbourhoods.