## *Structure of complex networks: Quantifying edge-to-edge relations by failure-induced flow redistribution*

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The analysis of complex networks has so far revolved mainly around the role of nodes and meaningful node communities. However, the dynamics of interconnected systems is commonly focalized on edge processes, and a dual edge-centric perspective can often prove more natural. To investigate the dynamical interplay between edges in a network, we present theoretical measures to quantify edge-to-edge relations.
We derive the flow-redistribution matrix, which describes a *topological property* of the network in the edge-space of the graph, inspired by the notion line failures in electrical circuits. We show how the flow-redistribution matrix can be decomposed into two measures with precise graph theoretic interpretations: the edge-to-edge transfer function, measuring the projection of the input flow in the cut-space of the graph, and the edge embeddedness, quantifying how strongly the edge features in the (weighted) cycles of the network. We apply these tools to reveal potentially *non-local* interactions between edges and showcase the general applicability of our edge-centric framework through constructive examples, as well as analyses of the Iberian Power grid, traffic flow in road networks, and the neuronal network of the nematode *C.elegans.*

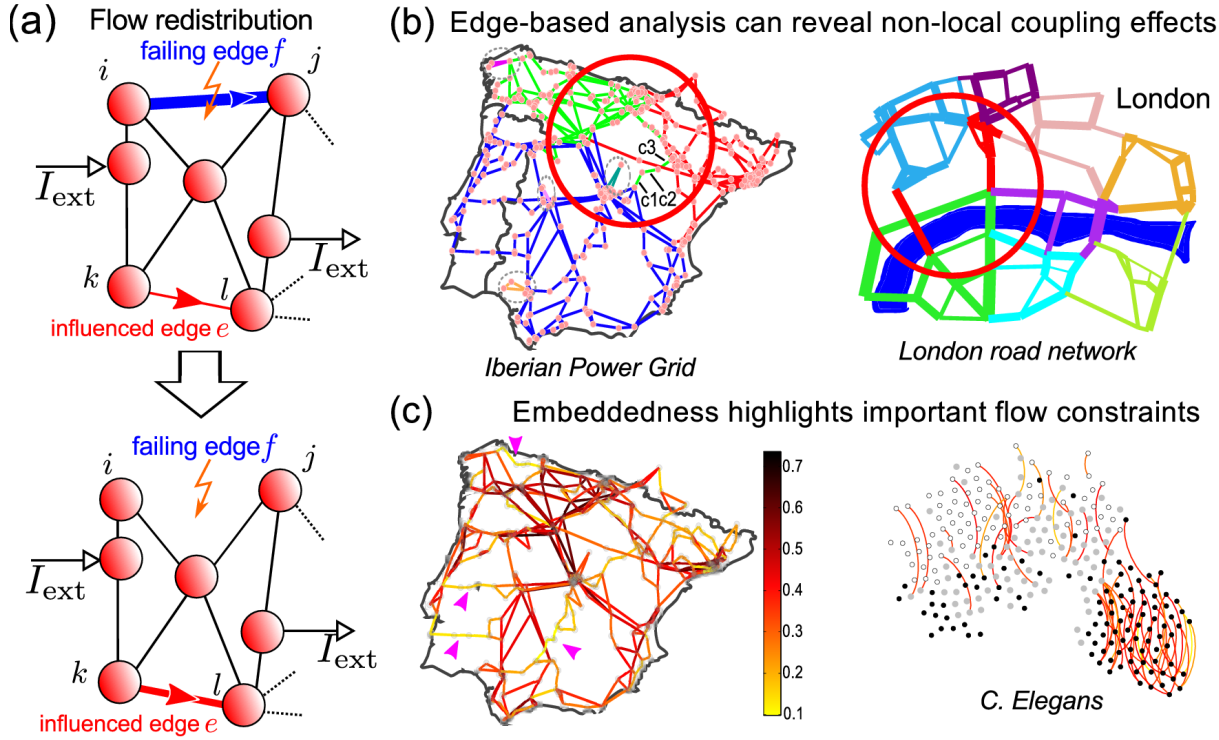


*Figure 1: (a) Schematic description of flow redistribution induced by a line failure: A line failure of edge f will influence the flow on other edges in the network, as illustrated here for edge e. (b) Communities of edges (denoted by different colours) found from the analysis of the flow-redistribution matrix. Note that the presence of non-local interactions effects between the edges is discovered. Left: the edges c1–c3 in the Iberian Power Grid are grouped with the north-west (green) community, although these edges lie between the north-east (red) and central-south (blue) communities and have no direct connection with the north-west (green) community. Small local circles (encircled with gray dotted lines), form their own isolated communities, i.e., they are effectively 'decoupled' from the rest of the network. A non-local interaction effect is also present in the road network of London (red circle). (c) Embeddedness analysis of edges. In the Iberian Power Grid our analyis reveals several weakly embedded paths of transmission lines (marked with magenta arrows), e.g., those connecting the center and south of Portugal with Spain; the lines going south from Madrid towards Andalusia; or the lines connecting Asturias and Galicia along the North-Northwest coast . For the C. Elegans neural network weakly embedded edges appear to be connecting mainly to motor neurons.*

**References**

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