Networks embedded in Lorentzian spaces

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Embedding the nodes of a network in a space with a metric can sometimes provide more natural or simple descriptions of the network's complex structure. Traditionally geometric approaches to network analysis have looked at networks embedded in spaces with a Riemannian metric, such as Euclidean space, or more recently hyperbolic space.

We will show how geometric techniques can also be used to approach networks that can be considered to be embedded in spaces with a Lorentzian metric, such as Minkowski space or De Sitter space. In particular where a natural time direction exists in a network (such as in citation networks, or task scheduling problems) or where edges correspond to a causal connection (Bayesian networks, or logic flows) the directed acyclic graph structure that results can be more naturally embedded with a Lorentzian metric where time is a coordinate treated differently to spatial coordinates.

We will firstly review simple models of Lorentzian embedded networks, such as Random Geometric Graphs under a Lorentzian geometry, and similar objects found elsewhere in physics such as the causal set approach to quantum gravity.

We will secondly then investigate how properties of the embedding space can be inferred from the network's structure, in both models and real data. For example, the dimension of that space is a measure of the network's global structure, and the coordinates of individual nodes in the embedding can reveal local information about the objects they represent. Importantly, many common network measures such as centrality, can be reinterpreted and adapted for the new constraints a Lorentzian embedding places on the network and so nodes which are 'central' by these new measures may be important in ways not previously recognised.

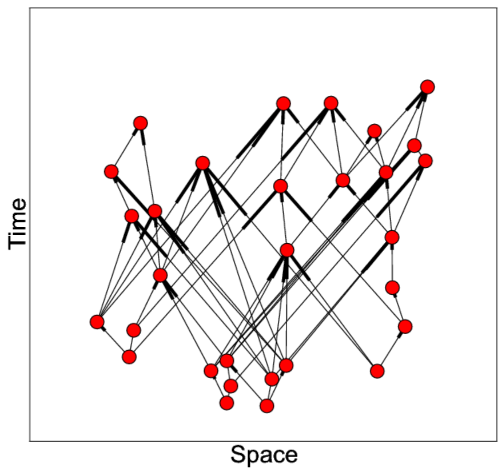

Figure 1: A diagram of a network embedded in a 2D Minkowski space, one of the simplest examples of a network showing a Lorentzian geometry.