What is the dimension of citation space?

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Why should we care about citation analysis?

- There's just too many papers too read
- We need ways of deciding which papers are likely to be useful to our research
- Citation analysis can provide a mechanism for quantifying this

Can we just count citations?

The simplest method of measuring usefulness is counting citations.
But not all citations mean the same thing

- Cite a paper because it was genuinely useful
- Cite their own paper
- Cite their colleague/friend's paper
- Reviewer inserts citation to their paper
- Author copies from the bibliography of another paper
- Cite well known paper in the field even if it was not useful to this work

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Is this a real problem?

- Academics and universities care about citation counts
- Journals care about impact factor
- Simkin & Roychowdhury estimated that around 80% of citations did not involve the author actually reading the paper they cite[2]

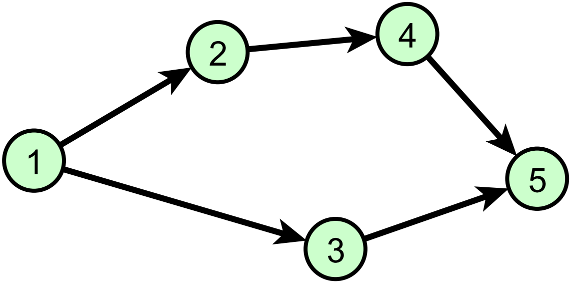
Solution? Use more of the network structure

- We have more information than just the number of citations a document has.
- There is a whole citation network structure to characterise and measure.
- Our approach - look at the causal structure of the network.

Citation Networks form Directed Acyclic Graphs

- We form a graph, where each document is a node
- A directed edge goes from node A to node B if A cites B in it's bibliography
- This means A must have been published *after* B, and edges go backwards in time. There can't be any closed loops (cycles).

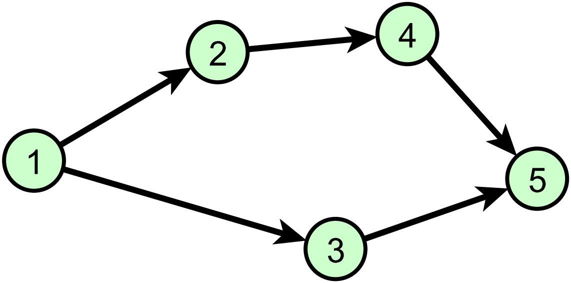
Citation Networks form directed Acyclic Graphs



Causal Structure

- Two nodes are causally connected if there is a path from one to the other, respecting edge direction.
- The set of these relations is what we mean by causal structure.

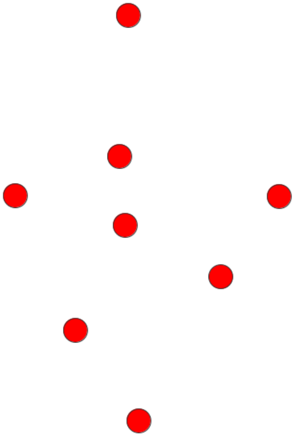
Causal Structure



Causal Structure

- We want to characterise the causal structure of a network
- We'll do this by making comparisons to the simplest set of models of networks with the same temporal constraints - networks embedded in Minkowski space.
- This model comes from a discrete approach to quantum gravity.

Take N nodes, and uniformly scatter them in a spacetime by giving them a time coordinate, t_α and $D-1$ spatial coordinates, x^i_α



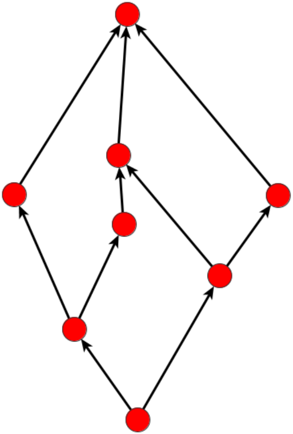
- Example of spacetime network where $D = 2$
- Time on vertical axis
- 1 spatial dimension on horizontal axis

- Take N nodes, and uniformly scatter them in a spacetime by giving them a time coordinate, t_α and $D-1$ spatial coordinates, x^i_α

- We then put an edge between two nodes, A and B if

$$(t_A - t_B)^2 > \sum_i (x^i_A - x^i_B)^2 \tag{1}$$

- So nodes are connected if they are more separated in time than they are in space
- Which is the same rule that defines how information can propagate through spacetime in special relativity.



- Example of spacetime network where $D = 2$
- Time on vertical axis
- $D - 1 = 1$ spatial dimension on horizontal axis
- Nearest neighbour links drawn for simplicity

Dimension

- Question - if we forget about the coordinates each point has, and just look at the nodes and edges can we work out what D was?

- Answer - yes - and this is how we will characterise these networks.

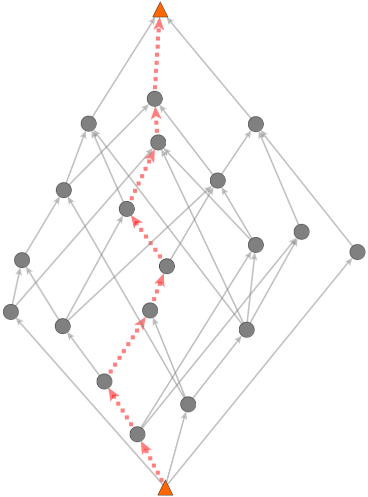
- Two ways of doing this developed in the causal set approach to quantum gravity. They only depend on the causal structure.

- While there are already methods of defining a ‘dimension’ for a network, they only consider spatial dimensions, but we will consider a time dimension separately as it has different constraints.

Method 1 - Midpoint Scaling Dimension

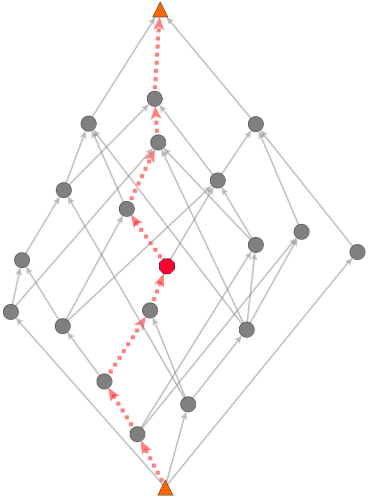
- Find a source node and a sink node - and look at the set of nodes in between them
- Find the longest chain from the source to the sink - this is a good approximation of the geodesic (shortest path) through that space

Method 1 - Midpoint Scaling Dimension



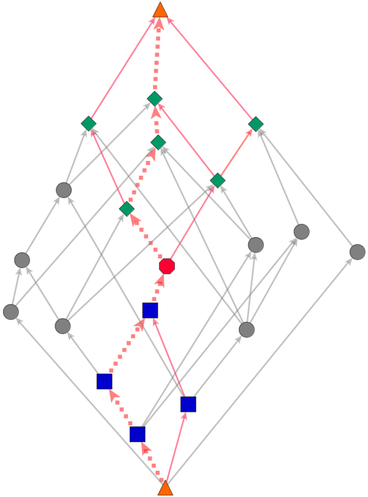
- Start and end nodes - orange triangles

Method 1 - Midpoint Scaling Dimension



- Start and end nodes - orange triangles
- Find midpoint - red octagon

Method 1 - Midpoint Scaling Dimension



- Start and end nodes - orange triangles
- Find midpoint - red octogon
- Find two intervals
- (start, middle) - blue squares
- (middle, end) - green diamonds
- The fraction of nodes in one of those intervals is $\frac{1}{2^D}$

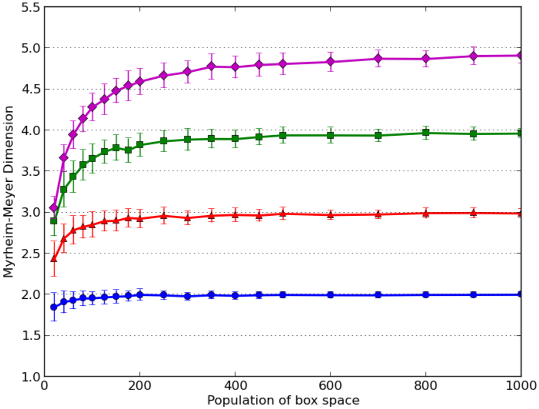
Method 2 - Myrheim-Meyer Dimension

Can be shown that the expected number of causally connected pairs, $\langle S_2 \rangle$ is just a function of N and D

$$\frac{\langle S_2 \rangle}{N^2} \equiv f(D) = \frac{\Gamma(D+1)\Gamma(D/2)}{4\Gamma(\frac{3}{2}D)} \tag{2}$$

So we can just measure how many of them there are, and then solve for D.[3]

OK - but does this actually work?



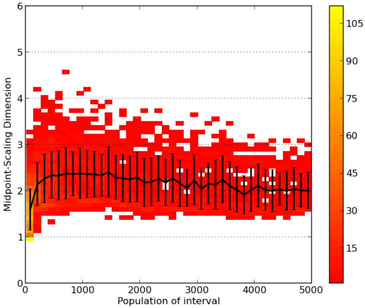
So what are we going to do?

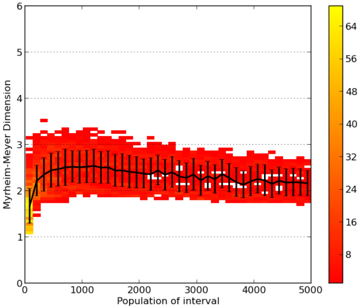
- So we have two different ways of measuring what kind of Minkowski space a network was embedded in
- We are now going to use these methods on real citation data and see what happens
- We will sample lots of intervals in the network to build up a picture of its causal structure

The data

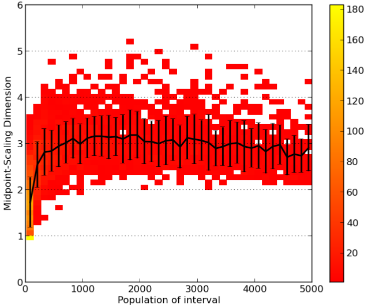
- Academic papers from the arXiv
- Patents from the USA (1970-2000)
- Judgements from the Supreme Court of the USA (1790-2012)

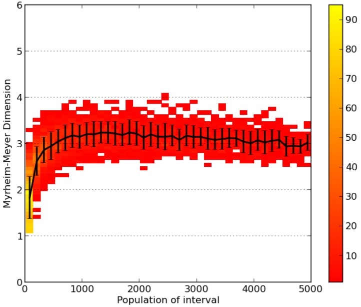
arXiv - high energy theory



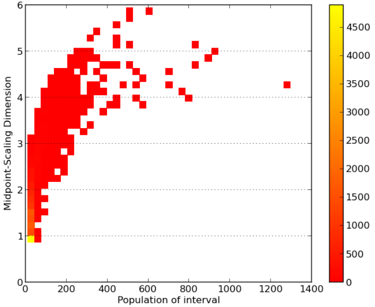


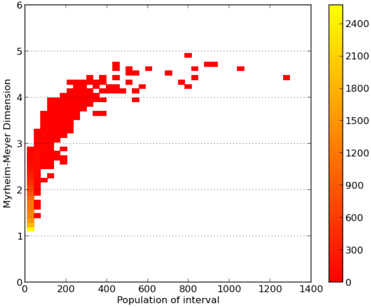
arXiv - high energy phenomenology



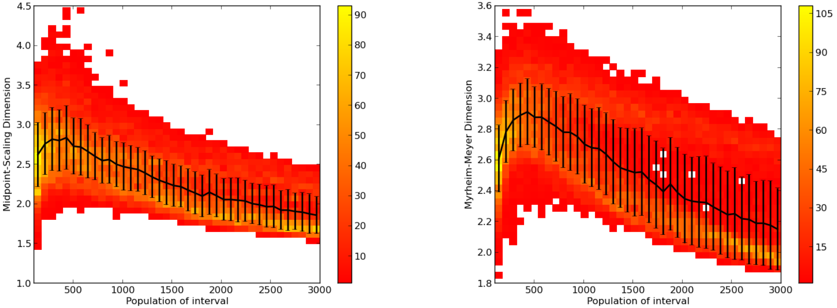


Patents


Midpoint-Scaling Dimension
0
1
2
3
4
5
6
0
200
400
600
800
1000
1200
1400
Population of interval
4500
4000
3500
3000
2500
2000
1500
1000
500
0


Myrheim-Meyer Dimension
0
1
2
3
4
5
6
0
200
400
600
800
1000
1200
1400
Population of interval
2400
2100
1800
1500
1200
900
600
300
0

Supreme Court



Interpretation

- High energy theory - 2
- High energy phenomenology - 3
- Patents - 5
- Supreme Court - 3 at small scales, 2 at large scales

Interpretation

- These dimension measures can easily distinguish between otherwise very similar citation networks
- They can be used to test whether models of citation network are really replicating the right behaviour on large scales

Interpretation

- We conjecture that these dimension measures can be interpreted in terms of how ‘broad’ or ‘narrow’ the citation behaviour in a field is.
- In a ‘narrow’ field where everybody cites all the same papers, the measured dimension would be 1
- In a ‘broad’ field where many people cite a paper without citing each other the measured dimension would be higher

Summary

- Independent dimension estimates agree, and there seems to be a consistently defined ‘spacetime dimension’ for citation networks
- They can be used to test whether models of citation network are really replicating the right behaviour, and can distinguish between similar networks
- Might help us ‘quantify interdisciplinarity’ - future work is on investigating this

Bibliography I

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What is the dimension of citation space?
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(Full list of citations available in this paper)

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Read before you cite!
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