How does mobility affect the connectivity and temporal correlation of interference in ad hoc networks?

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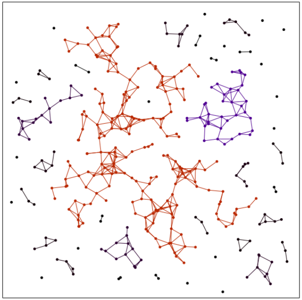
Motivation

Mobility induces inhomogeneity in Mobile Ad Hoc Networks. When does this inhomogeneity favour/hinder us?

We consider two approaches:

- Single snapshot coverage - inhomogeneous distribution
- Temporal correlation of interference under the Random Waypoint Mobility Model



Outlook

1. General model
2. Nearest Neighbour Association
3. Coverage in Inhomogeneous networks
4. Temporal correlation of interference with mobility
5. Impact of blockages on temporal correlation with node mobility

Model:Single time slot analysis

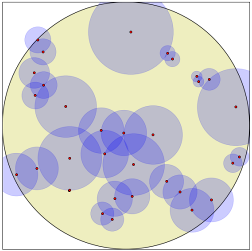
Inhomogeneous Poisson Point Process, $\lambda(r)$, on a Disk $\mathcal{V}$.
Nearest Neighbour Association model- with pdf $f(\boldsymbol{y}, d_1)$.
Probabilistic Connection Model that depends on small and large scale fading, and assuming equal transmit powers:

$$\mathcal{H}_1 = \mathbb{P}\left[\frac{g(d_1)|h_1|^2}{\mathcal{N} + \gamma \sum_{k>1} g(d_k)|h_k|^2} \geq q|\boldsymbol{y}, d_1\right]$$

where pathloss $g(x) = x^{-\eta}$

Coverage Probability: Probability that a receiver can decode a message from its nearest transmitter [Banani15]

$$C(\boldsymbol{x}) = \int_0^{d_{max}} f(\boldsymbol{y}, d_1)\mathcal{H}_1(\boldsymbol{y}, d_1)\mathrm{d}d_1$$



Set up

Inhomogeneous Poisson Point Process on a Disk,

$$\lambda(r) = \lambda_0(a + br^2), \quad \text{where} \quad 2\pi \int_0^R \lambda(r) r \mathrm{d}r = \lambda_0 |\mathcal{V}|$$

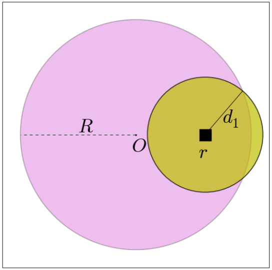
The complementary cdf of d_1, is the probability that there are no points within $B_{\boldsymbol{y}}(d_1)$

$$\bar{F} = \mathbb{P}[N(B_{\boldsymbol{y}}(d_1)) = 0] = e^{\left(-\int_{B_{\boldsymbol{y}}(d_1)\cap\mathcal{V}} \lambda(\boldsymbol{y})\mathrm{d}\boldsymbol{y}\right)}$$

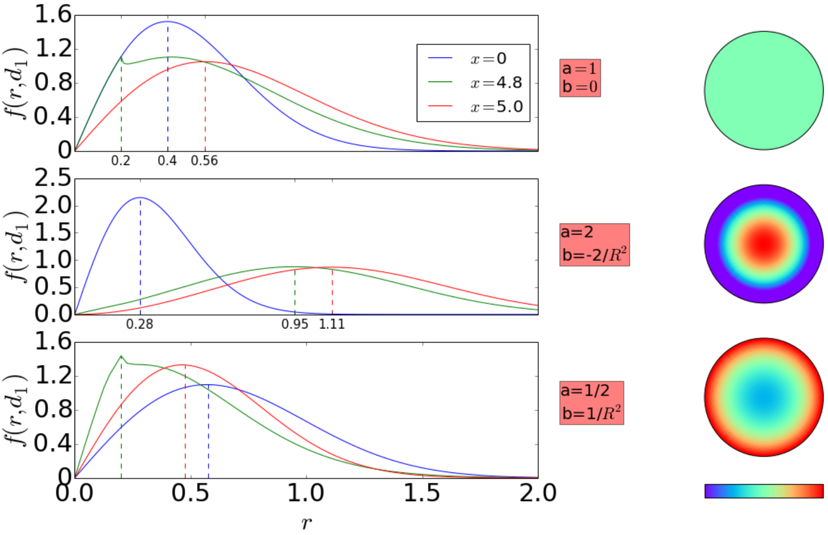
Thus the Nearest neighbour distribution (NND) is defined as,

$$f(r, d_1) = \frac{\mathrm{d}}{\mathrm{d}d_1}(1 - \mathbb{P}\left[N(B(\boldsymbol{y}, d_1)) = 0\right])$$

$$= -\frac{\mathrm{d}}{\mathrm{d}d_1} \exp\left(-\int_{B_{\boldsymbol{y}}(d_1)\cap\mathcal{V}} \lambda(\boldsymbol{y})\mathrm{d}\boldsymbol{y}\right)$$



Nearest Neighbour Distribution



Connection Probability

$\mathcal{H}_1(d_1, \boldsymbol{y})$: The probability that a receiver at $\boldsymbol{y}$ can successfully decode a signal from its nearest neighbour separated by a distance d_1.

$$\mathcal{H}(d_1, \boldsymbol{y}) = \mathbb{P}[\mathrm{SINR}_1 \geq q \big| \boldsymbol{y}, d_1] = \mathbb{P}\left[\frac{|h_1|^2 g(d_1)\mathcal{P}_1}{\mathcal{N} + \gamma \sum_{k>1} |h_k|^2 g(d_k)\mathcal{P}_k}\right]$$

$$= \mathbb{P}\left[|h_1|^2 \geq \frac{q(\mathcal{N} + \gamma \sum_{k>1} \mathcal{I}_k)}{\mathcal{P}g(d_1)}\Big| \boldsymbol{y}, d_1\right]$$

$$\mathcal{H}(d_1, \boldsymbol{y}) = \exp\left(-\frac{q\mathcal{N}}{\mathcal{P}g(d_1)}\right)\mathcal{L}_{\mathcal{I}_1}(s_1)$$

where $s_1 = \frac{q\gamma}{g(d_1)}$ and $\mathcal{L}_{\mathcal{I}_1}(s_1)$ is the Laplace transform of the r.v $\mathcal{I}_1$ at s_1

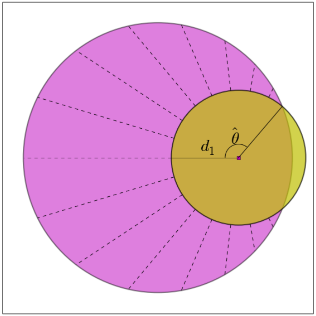
$\mathcal{L}_{\mathcal{I}_1}(s_1) = \exp\left[-\int_{\mathcal{V}\setminus B_y(d_1)} \frac{s_1 g(d_k)}{1+s_1 g(d_k)} \Lambda(\mathrm{d}d_k)\right]$ [Haenggi13]

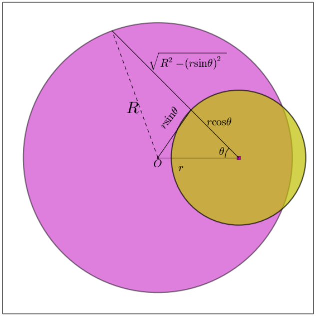
Connection Probability

$$\mathcal{L}_{\mathcal{I}_1}(s_1) = \exp\left[-\int_{\color{red}{\mathcal{V}\setminus B_y(d_1)}} \frac{s_1 g(d_k)}{1+s_1 g(d_k)}\,\Lambda(\mathrm{d}d_k)\right]$$

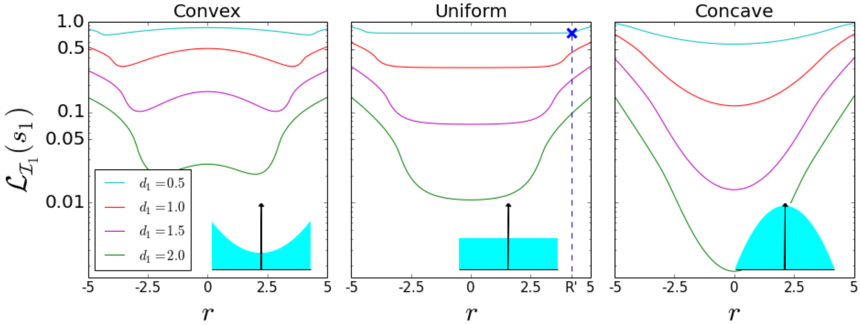
$$= \exp\left[-2\int_0^{\hat{\theta}_1}\int_{d_1}^{R_{max}} \frac{d_k}{1+\frac{1}{s_1 g(d_k)}}\lambda\left(\sqrt{d_k^2+r^2-2d_k r\cos\theta}\right)\mathrm{d}\theta\,\mathrm{d}d_k\right]$$

where; $\hat{\theta}_1 = \min\left[\left|\arccos\left[\frac{r^2+d_1^2-R^2}{2xd_1}\right]\right|, \pi\right]$ and $R_{max} = r\cos\theta + \sqrt{R^2 - r^2\sin^2\theta}$





The Laplace transform of the random variable $\mathcal{I}_k$ evaluated at $s_1 = \frac{q\gamma}{g(d_1)}$ for a Disk.



Parameters: $R = 5$; $\eta = 6$;$\mathcal{P} = 1$; $q = 1$; $\gamma = 1$

Network Coverage

The probability that a receiver located at $\boldsymbol{y}$ is in downlink of its nearest transmitter.

$$C(\boldsymbol{y}) = \int_0^{d_{max}} f(\boldsymbol{y}, d_1)\mathcal{H}_1(d_1, \boldsymbol{y})\,\mathrm{d}d_1$$

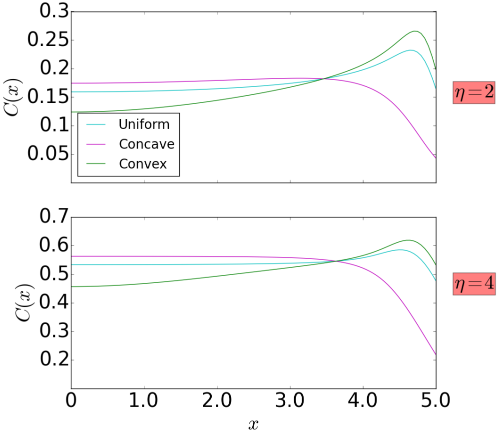
$\lambda_0 = 1$; $R = 5$

Border effects: Deterioration in performance

Poor performance in areas of high node density, due to high proximity of interferes

Convex case benefits from higher node density near border

$\eta = 4$: is better than $\eta = 2$:



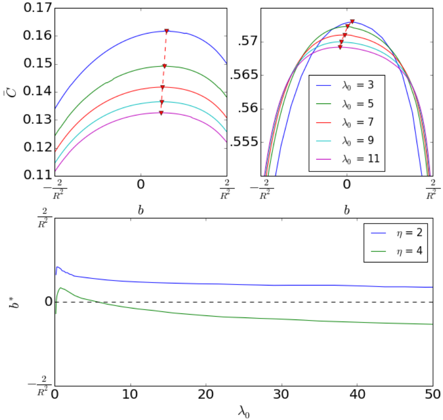
Optimal Distribution of Transmitters

Define the average coverage for a uniform distribution of receivers,

$$\bar{C} = \frac{2}{R^2} \int_0^R C(r) r \mathrm{d}r$$

Find b^* such that $\bar{C}$ is maximised,

$$b^*(\eta, \lambda_0 \pi R^2) = \arg\max_b \bar{C}$$



Temporal Correlation of Interference

Calculation of interference

$$\mathcal{I}(t) = P_t \sum_i \xi_i(t)\, h_i(t)\, \beta_i(t)\, g\left(x_i(t) - y_p\right)$$

Temporal correlation of interference

- arises due to correlated traffic, MAC scheme, correlated propagation channel and user mobility
- has an impact on the outage, the diversity, the local delay, multi-hop delay, etc.

Pearson correlation coefficient at time lag $l = |t - \tau|$ in the steady state

$$\rho_l = \frac{\mathbb{E}\left\{\mathcal{I}(t)\,\mathcal{I}(\tau)\right\} - \mathbb{E}\left\{\mathcal{I}(t)\right\}^2}{\mathbb{E}\left\{\mathcal{I}^2(t)\right\} - \mathbb{E}\left\{\mathcal{I}(t)\right\}^2}$$

Blockage and finite borders introduce correlation in the interference levels between the users over space and time

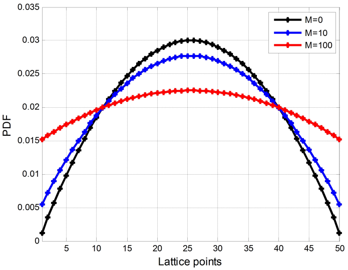
- transitions between LoS and NLoS propagation conditions due to mobility will reduce the temporal correlation of interference

System model

Assuming a random access MAC the temporal correlation of interference depends on

1. the steady state distribution of users
2. the user displacement law
3. the correlation in the channel due to blockage and finite borders

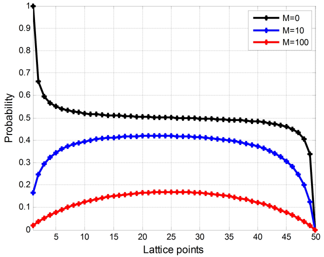
For RWPM model over 1D lattice $n = 1, 2, \ldots N$ and constant speed u for all users $f_{x_i}(n) = \frac{p}{N} + (1-p)\frac{3N(2n-1)-6n(n-1)-3}{N(N^2-1)}$, $p = \frac{M/2}{M/2+(N+1)/(3u)}$



User displacement law

Given that the user is located at lattice point n at $t = 0$, the displacement law is essentially the probability $\mathbb{P}(n+k,\tau)$ that the user moves to lattice point $(n+k)$ at $t = \tau$

- The probability that the user remains static at $t = 1$ is location-dependent, $\mathbb{P}(n,1) = \frac{p}{Nf_x(n)}$
- The probability that the user moves right is equal to the probability that the user moves times the percentage of paths crossing the lattice point n and moving right, $\mathbb{P}(n+1,1) = \frac{(1-\mathbb{P}(n,1))n(N-n)}{n(N-n)+(n-1)(N-n+1)}$



Blockage model

- Poisson number of blockage $\mathrm{Po}(\alpha)$ with fixed but unknown locations over continuous domain $[1, N]$
- penetration loss per obstacle follows the uniform distribution $U(0, \gamma)$
- blockages do not hinder the user moves

Moments of the penetration loss β over distance d

$$\mathbb{E}(\beta^s) = e^{-\alpha d \left(1 - \frac{1}{1+s}\gamma^s\right)}$$

The cross-correlation of penetration losses from lattice points n, m depends on their relative location w.r.t. the point where interference is computed

$$\mathbb{E}\{\beta_n \beta_m\} = e^{-\alpha(d_n + d_m)\left(1 - \frac{\gamma}{2}\right)}$$

$$\mathbb{E}\{\beta_n \beta_m\} = e^{-\alpha\left(\min\{d_n, d_m\}\left(1 - \frac{\gamma^2}{3}\right) + |d_m - d_n|\left(1 - \frac{\gamma}{2}\right)\right)}$$

Interference moments

Assuming Poisson number of users Po(K) with i.i.d. activity $\xi \leq 1$, unit-mean Rayleigh fading and distance-based propagation pathloss $g(\cdot)$, the mean of interference is

$$\mathbb{E}\{\mathcal{I}\} = \xi \sum\nolimits_n e^{-\alpha d_n \left(1-\frac{\gamma}{2}\right)} g(d_n) f_x(n)$$

The second moment of interference depends on the correlated slow fading

$$\mathbb{E}\{\mathcal{I}^2\} = 2K\xi \sum\nolimits_n e^{-\alpha d_n \left(1-\frac{1}{3}\gamma^2\right)} g^2(d_n) f_x(n) + K^2 \xi^2 \sigma$$

where $\sigma = \sum_{n,m} \mathbb{E}\{\beta_n \beta_m\} g(d_n) g(d_m) f_x(n) f_x(m)$

The cross-correlation of interference depends on the user displacement law and the correlation of the RVs $\beta_i(t)$ and $\beta_j(\tau)$

$$\mathbb{E}\left\{\mathcal{I}(t)\mathcal{I}(\tau)\right\} = K\xi^2 \sigma_I + K^2 \xi^2 \sigma$$

where

$$\sigma_I = \sum\nolimits_{n,k} \mathbb{E}\{\beta_n \beta_{n+k}\} g(d_n) g(d_{n+k}) \mathbb{P}(n+k, \tau) f_x(n)$$

Properties

- Without blockage, i.e., $\alpha = 0$, the correlation coefficient ρ_I is independent of the user density. Also, $\rho_I|_{\alpha=0} \leq \frac{\xi}{2}$.
- With blockage, the correlation coefficient ρ_I increases with the number of users K.

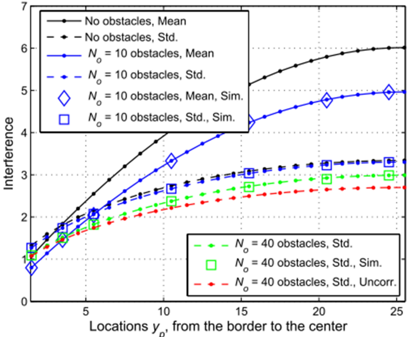
If we expand ρ_I around $K \to \infty$, we get $\rho_I = 1 - \frac{c_3 - c_1}{K c_2} + \mathcal{O}\left(\frac{1}{K}\right)^2$ where $c_1 = \xi \sigma_I$, $c_2 = \xi \sigma - \xi (\sum_n \mathbb{E}\{\beta_n\} g(d_n) f_x(n))^2$, $c_3 = 2 \sum_n \mathbb{E}\left\{\beta_n^2\right\} g^2(d_n) f_x(n)$. Therefore for a large K, $\rho_I > \frac{\xi}{2} \geq \rho_I|_{\alpha=0}$.

For $K \to 0$, one can show that $\rho_I = \frac{c_1}{c_3} \leq \rho_I|_{\alpha=0}$ using that $\mathbb{E}\left\{\beta_n \beta_{n+k}\right\} \leq \mathbb{E}\left\{\beta_n^2\right\} \forall \{n, k\}$.

- In mobile wireless networks, blockage reduces the correlation of interference at low user densities, while the opposite is true at high user densities

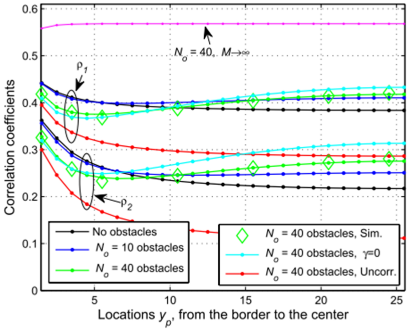
Numerical Examples I

- The impact of blockage on the mean interference is more prominent close to the center of the lattice
- Near the boundaries fewer users are located and the interference is practically generated from one direction
- Blockage increases the variance of interference
- Ignoring the spatial correlation of users due to blockage results in non-negligible error for the standard deviation



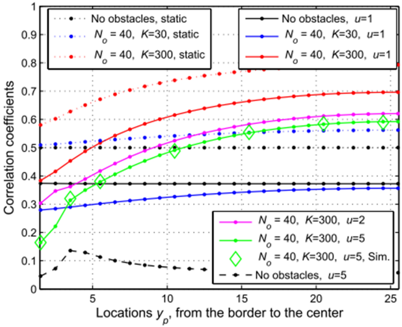
Numerical Examples II

- Close to the boundaries, where the user density is low, the transitions from LoS to NLoS and vice versa dominate, and the interference correlation becomes less as compared to the case without obstacles.
- Close to the center, where the user density is high, the correlated interference levels from the different users dominate over the randomness introduced by the mobility, and the correlation coefficients become higher.



Numerical Examples III

- Increasing user speeds in environments with blockage do not help much in reducing the temporal correlation of interference when the user density is high
- Taking the limit at $u \to \infty$ can be used to show that interference correlation stays positive even if the user locations are uncorrelated over time



Conclusion

1. User coverage depends on location, distribution of transmitters, border affects and the pathloss model.
2. Correlated slow fading due to blockage, user density and mobility have a joint impact on the temporal correlation of interference

Application

1. Network densification means adaptive routing protocols are needed to maximise user coverage.

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