Stability and rate of convergence of resource allocation within a cellular network

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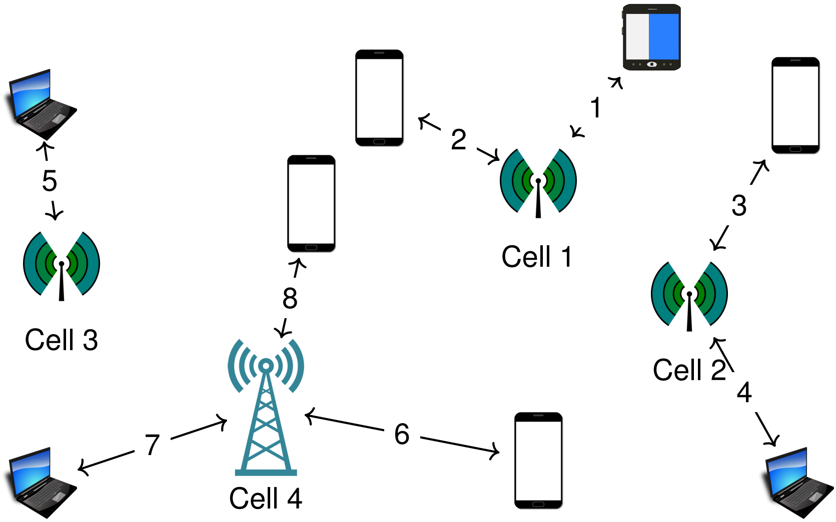
Presentation outline

- Definitions
- RB allocation
- Equations
- Existence of a solution
- Formulation 1: Newton's method
- Formulation 2: continuous approximation
- Formulation 3: least squares error
- Comparisons
- Self-organising networks

Example network



Definitions 1/3

Notation

Number of UEs = n
Number of cells = m.

I define the following sets

- C denotes the set of all cells.
- U denotes the set of all UEs.
- $Q := \{1, \ldots, \mathrm{n}\}$
- $U|c \subset U$ is the set of all UE's connected to cell c.

Definition

The proportion of the bandwidth allocated to u_j is $\mathbf{x}_j$.

Definitions 2/3

Notation

Let $u_j \in U$ be connected to a cell then I will denote that cell by $c_{\bar{\jmath}} \in C$, otherwise I will denote a general cell by $c \in C$

- $\mathbf{S}_j$ denotes the SINR from cell $c_{\bar{\jmath}}$ to UE u_j.
- I_j is the unwanted signals in the channel between $c_{\bar{\jmath}}$ and u_j, in Watts.
- σ^2 is the internal noise that is added to the system, in Watts.
- Ψ_j is the signal strength of u_j and $c_{\bar{\jmath}}$ connection, in Watts.

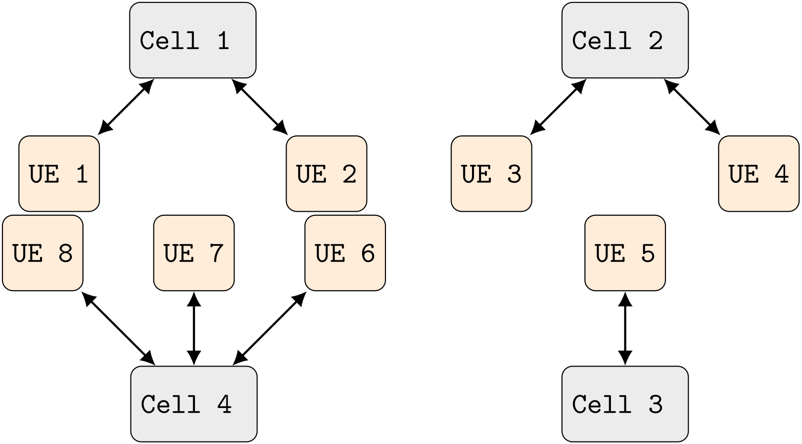
Definitions 3/3

Notation

$P \in \mathbb{R}^{n \times m}$ is the pathloss matrix between the UEs and cells.
$A \in \{0, 1\}^{n \times m}$ is the cell-to-UE mapping matrix.

$$P = \begin{pmatrix} 0 & p_{1,2} & p_{1,3} & p_{1,4} \\ 0 & p_{2,2} & p_{2,3} & p_{2,4} \\ p_{3,1} & 0 & p_{3,3} & p_{3,4} \\ p_{4,1} & 0 & p_{4,3} & p_{4,4} \\ p_{5,1} & p_{5,2} & 0 & p_{5,4} \\ p_{6,1} & p_{6,2} & p_{6,3} & 0 \\ p_{7,1} & p_{7,2} & p_{7,3} & 0 \\ p_{8,1} & p_{8,2} & p_{8,3} & 0 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Abstract network diagram.



RB allocation in one timeslot

Cell 1	1 **UE 1**	2	3	4 **UE 2**	5	6	7
Cell 2	1	2 **UE 3**	3 **UE 3**	4	5 **UE 4**	6 **UE 4**	7 **UE 4**
Cell 3	1 **UE 5**	2 **UE 5**	3 **UE 5**	4 **UE 5**	5	6	7
Cell 4	1 **UE 6**	2	3 **UE 7**	4 **UE 7**	5	6 **UE 8**	7 **UE 8**

Frequency →

Probability of resource block collision

Notation

$\mathbb{P}(u_j \leftrightarrow c)$ is the probability that a resource block is assigned to u_j by $c_{\bar{j}}$ and is also assigned to some other UE by cell c.

I assume that the resource blocks are allocated uniformly at random across the entire bandwidth. Hence:

$$\begin{aligned}\mathbb{P}(u_j \leftrightarrow c) = & \text{Proportion of bandwidth assigned to } u_j \quad \times \\ & \text{Proportion of bandwidth assigned by } c\end{aligned}$$

$$\mathbb{P}(u_j \leftrightarrow c_i) = \mathbf{x}_j(A\mathbf{x})_i$$

Interference

The unwanted signal generated by a cell c_i which interferes with the channel between u_j and $c_{\bar{\jmath}}$ is given by:

$$\text{Intf}(u_j, c_i) = \text{Channel gain}(u_j, c_i) \times \text{Prob. of collision}$$

Then the total inference from other cells received by UE u_j is the sum of the interferences.

$$I_j := \mathbf{x}_j \sum_{i=1}^{m} P_{i,j}(A\,\mathbf{x})_i$$

SINR

Notation

The symbols $\odot$ and $\oslash$ denote the Hadamard product and division respectively.

The signal to noise + interference vector is given by:

$$\mathbf{S}(\mathbf{x}) = \Psi \oslash (\mathbf{x} \odot PA\mathbf{x} + \sigma^2)$$

SINR to spectral efficiency

The spectral efficiency is the channel capacity per unit bandwidth.

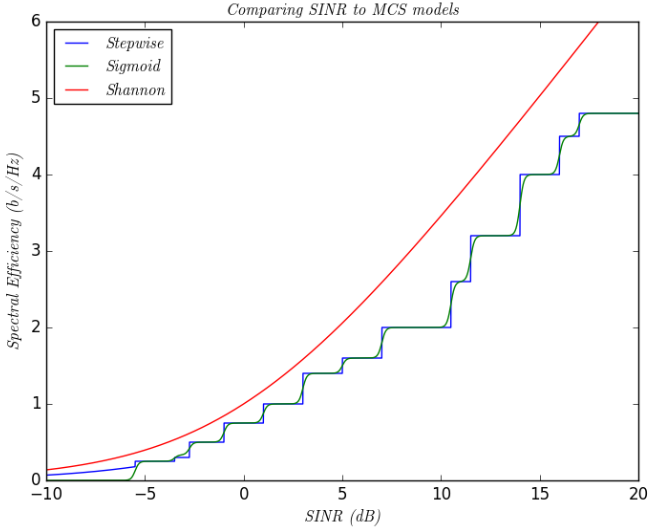
Units

Spectral efficiency has units $b/s/Hz = b$, which is a counting variable and hence dimensionless.

I denote function $f : \mathbb{R}^n \to \mathbb{R}$ to be the single spectral efficiency function. I then vectorise this into the function $F : \mathbb{R}^n \to \mathbb{R}^n$ which I define to be:

$$F(\mathbf{S}) = [f(\mathbf{S}_1), \ldots, f(\mathbf{S}_n)]^T$$

SINR to spectral efficiency



Optimisation

I define the vector of data demands to be **d**. I seek the solution to the fixed point problem:

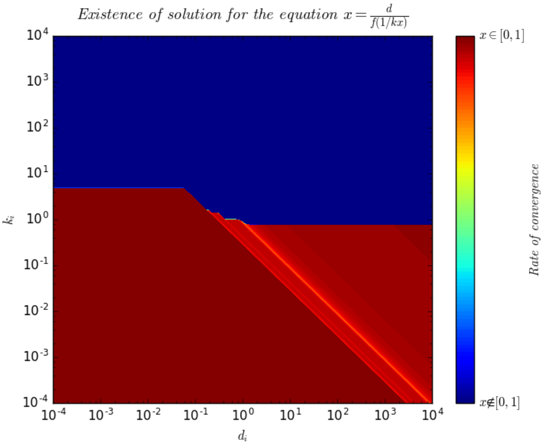
$$\mathbf{x} = \mathbf{d} \oslash F(\Psi \oslash (\mathbf{x} \odot PA\mathbf{x} + \boldsymbol{\sigma}^2)) \qquad \mathbf{x} \in (0,1]^n$$

This can be written as a iterative method:

$$\mathbf{x}_{t+1} = \mathbf{d} \oslash F(\Psi \oslash (\mathbf{x}_t \odot PA\mathbf{x}_t + \boldsymbol{\sigma}^2))$$

Using this formula in conjunction with the trust region method, I will refer to the autonomous simulation.

Existence of a solution



Parameters

- f - sigmoid
- $\sigma^2 = 0$
- $k = PA\mathbf{x} \oslash \Psi$

Existence of a solution

Consider the Shannon spectral efficiency, using bounds on logarithms I define a sufficient condition for non-existence.

If $\exists i \in Q$ such that:

$$\Psi_i \geqslant \left(\sqrt{\frac{\Psi_i}{\mathbf{d}_i (PA\mathbf{x})_i \ln(2)}} - 1\right)\left(\mathbf{x}_i (PA\mathbf{x})_i + \sigma^2\right)$$

Then there does not exist a solution.

Formulation 1: Newton's method

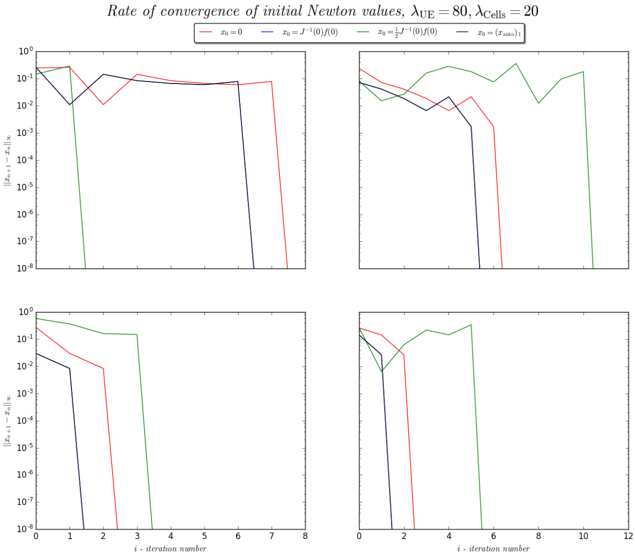
We seek the root of the following equation using Wolfe conditions and a trust region:

$$G(\mathbf{x}) = \mathbf{x} \odot F(\Psi \oslash (\mathbf{x} \odot PA\,\mathbf{x} + \sigma^2)) - \mathbf{d} \quad \mathbf{x} \in (0,1]^n$$

The Jacobian is given by:

$$J_G(\mathbf{x})_{i,j} := \begin{cases} f(\mathbf{S}_i(\mathbf{x})) - (PA\,\mathbf{x})_i \left(f'(\mathbf{S}_i(\mathbf{x})) \frac{\mathbf{S}_i^2\,\mathbf{x}_i}{\Psi_i} \right) & \text{if } i = j \\ -PA_{i,j} f'(\mathbf{S}_j(\mathbf{x})) \frac{\mathbf{S}_j^2\,\mathbf{x}_j^2}{\Psi_j} & \text{if } i \neq j \end{cases}$$

Formulation 1: Newton's method



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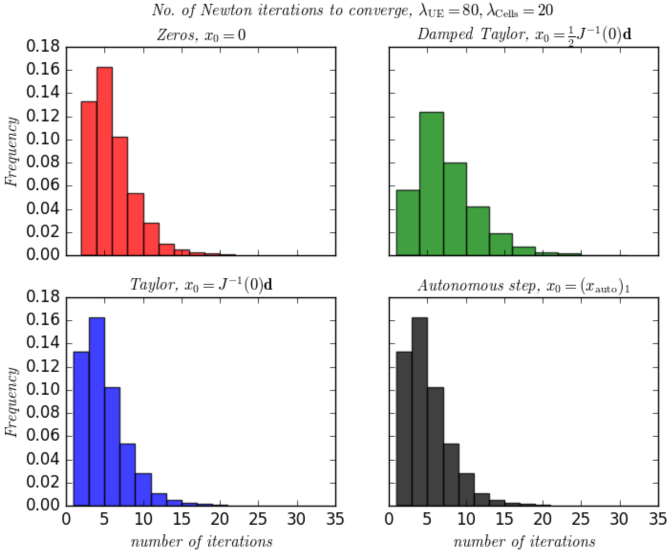

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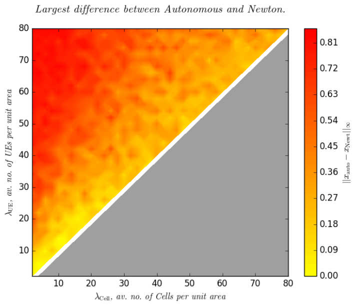
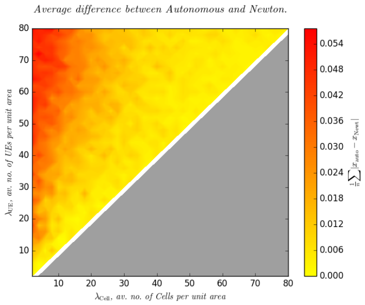




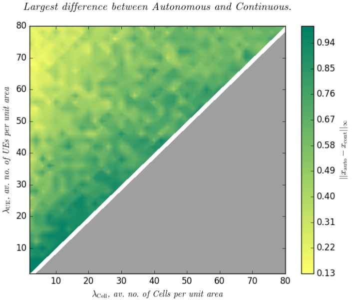
Formulation 2: Continuous approximate

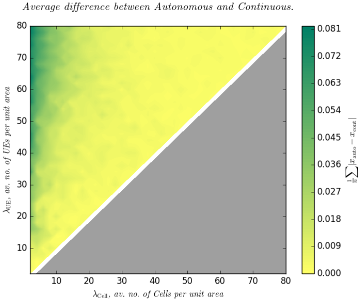
Using an approximation, I am able express the problem in terms of an autonomous ODE.

$$\dot{\mathbf{x}} = \mathbf{d} \oslash F(\Psi \oslash (\mathbf{x} \odot PA\mathbf{x} + \sigma^2)) - \mathbf{x}$$

This has no exact solution and hence I use Runge-Kutta methods to solve this system.

Formulation 2: Continuous approximate





Formulation 3: Least squares error

I wish to minimise the proportion of bandwidth allocated to the UEs such that the data demands are met.

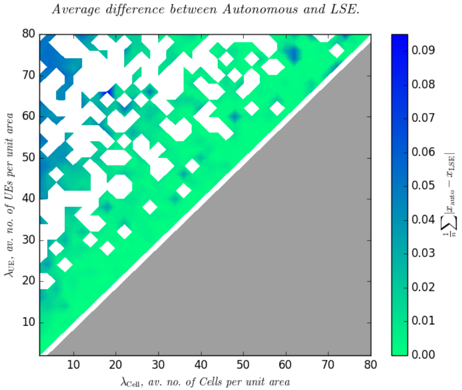
$$\mathbf{x}^* = \mathrm{argmin}(||\,\mathbf{x}\,||_2) \text{ such that } G(\mathbf{x}^*)_i \geqslant 0, \mathbf{x}^* \in [0,1]^n, \forall i \in Q$$

If the Shannon spectral efficiency is being considered then $G(\mathbf{x})$ is a concave function and hence feasible region is convex. Therefore this problem may be solved as a convex optimisation problem.

Formulation 3: Least squares error

Problem

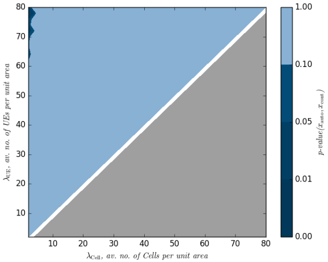
The least squares model makes the assumption that all of the data demands can be met, that is usually not the case.



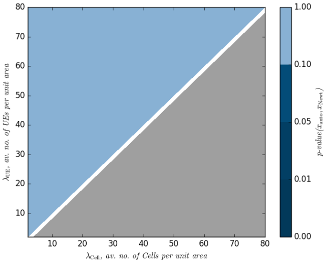
Comparisons

I wish to consider the accuracy of these algorithms, hence I performed the Kolmogorov-Smirnov test to compare the distributions of the proportion of resource blocks allocated.

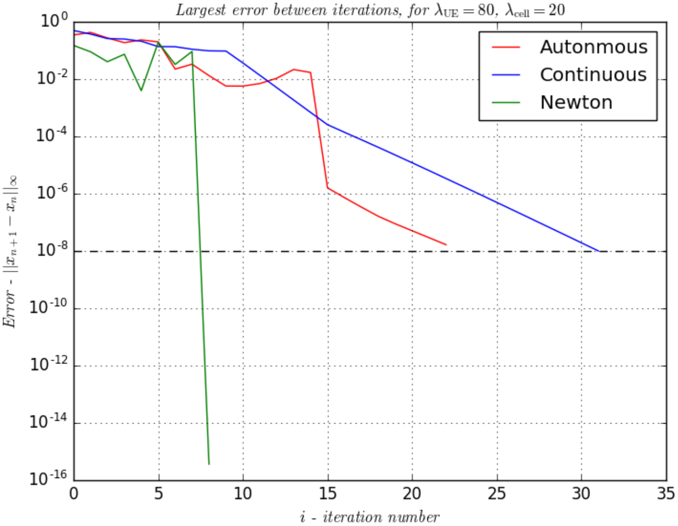
Average K-S test p-value between Autonomous and Continuous.



Average K-S test p-value between Autonomous and Newton.



Comparisons



Self-organising networks

Implementing a Newton method on clusters within the network would be the best way to have the network be self organising, due to the speed of convergence.The information that would need to be shared.

- Proportion of the bandwidth that each cell assigns to a UE.
- The positions of each cell (Fixed)
- The positions of each UE (Dynamic)
- The data demands of each UE.

Thank you for your time.