Mixed and time varying models for evolving complex networks

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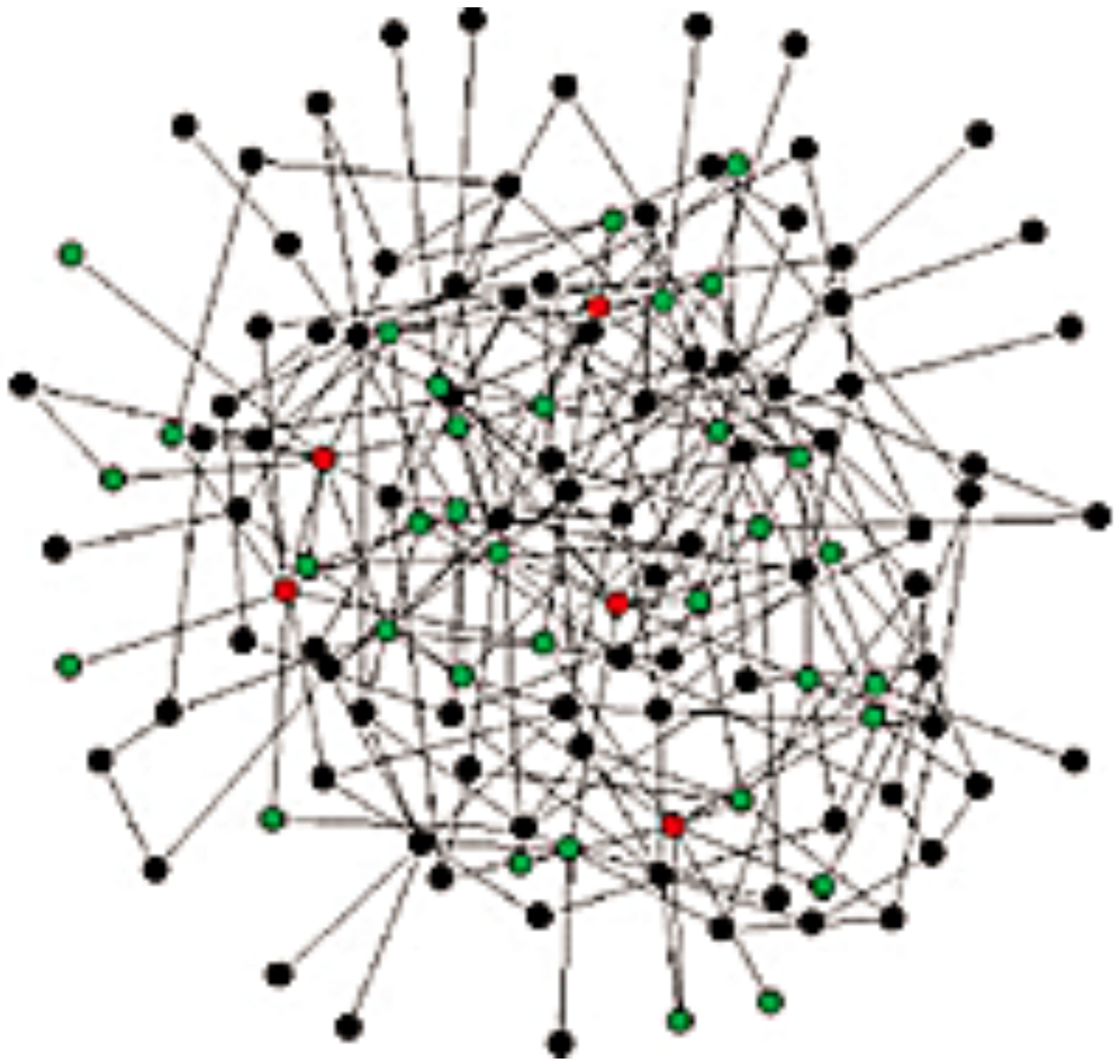
Outline

1. Background
2. Mixed models
3. Time varying models
4. Future directions

Generative models for network structure: Static



- Generates single network snapshots
- Examples: Erdős–Rényi, Stochastic Block Model, Configuration Model, ERGMs
- Community detection, centrality

Generative models for network structure: Dynamic

- Generates network by addition of nodes and links
- Investigate how network structural features emerge

Example: Barabási-Albert model

Characterised by:

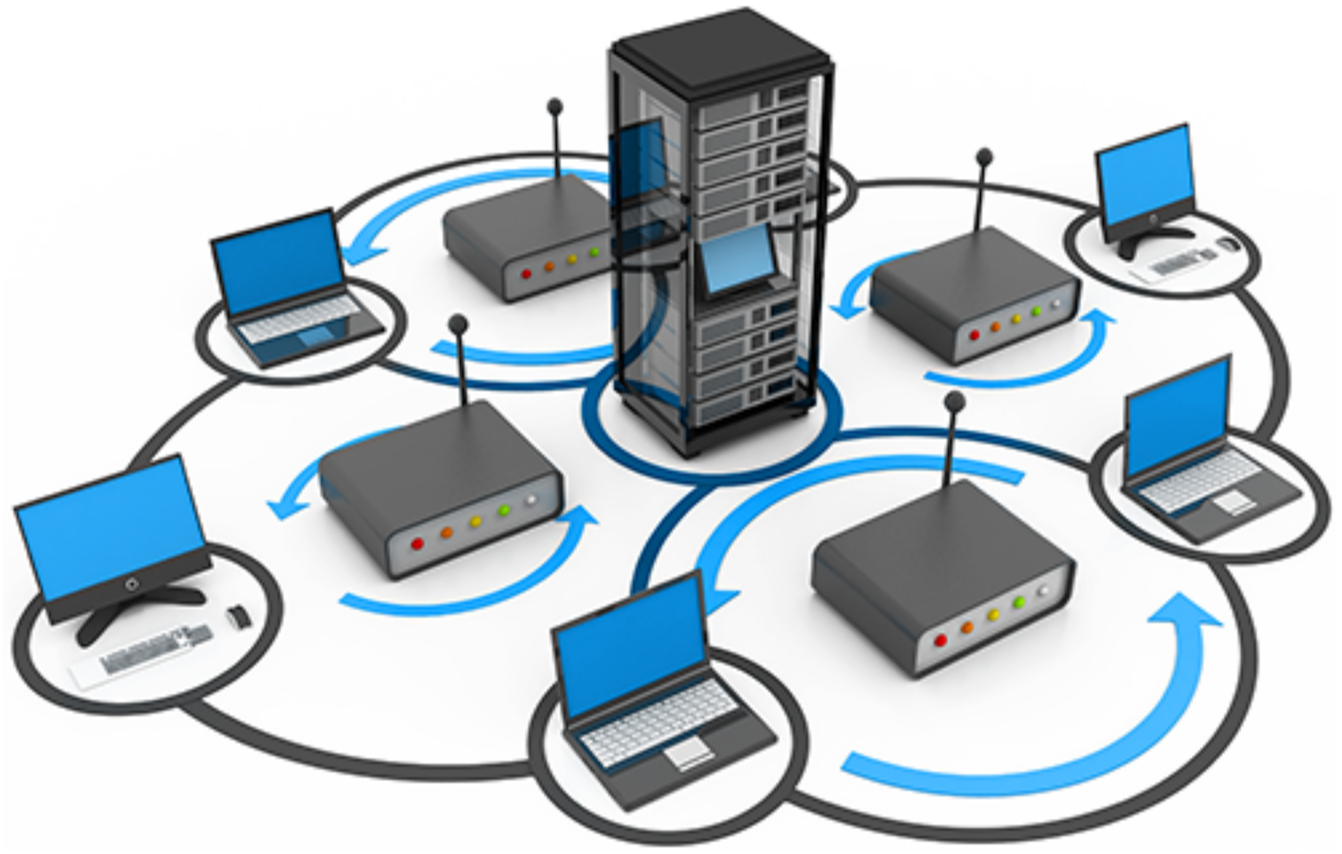
<u>Growth</u>: network grows by iterative addition of a single node with m neighbours

<u>Preferential attachment:</u> new node connects to existing node i with probability $\propto k_i$.

i.e. better connected nodes likelier to attract new links

$m = 2$

Examples of evolving networks



Usual modelling assumptions

- Network grows using a single constant mechanism
- Good for deriving theoretical results…
- … but not great for making inferences from real data

Our aim

To relax the usual modelling assumptions made, to better comprehend a model governing a network’s evolution

“Single mechanism” → more than one mechanism, to uncover the roles of each of them.

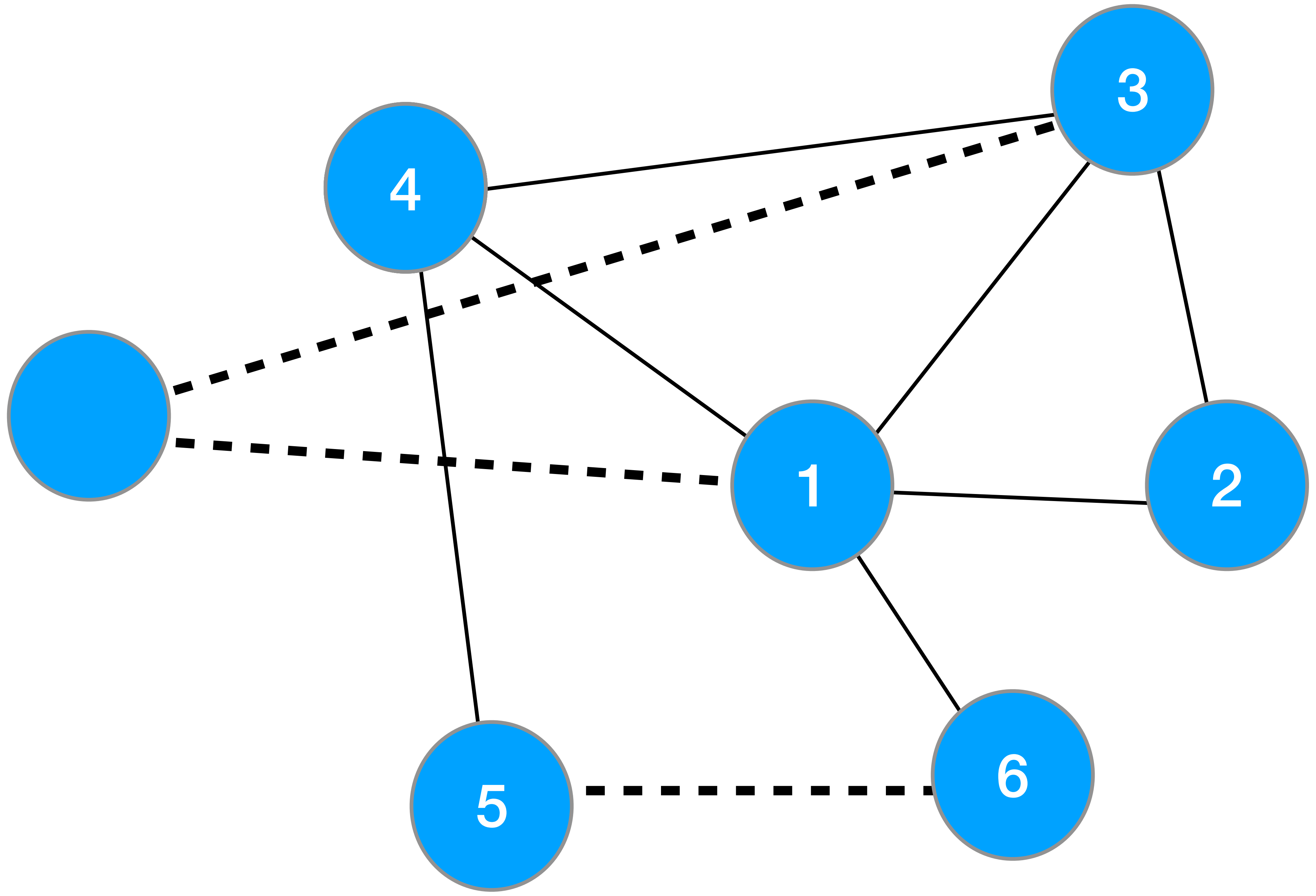
“Constant” → changing in time, to understand how these roles may change over time

Model for evolving networks

Seed network

Attachment probabilities

Action (new node/internal link)



$$p_i$$

corresponding to

$\mathbb{P}($**choose node** $i)$

Attachment probabilities

$$p_i \propto 1$$

Random/neutral model. All nodes equally likely

$$p_i \propto f(k_i)$$

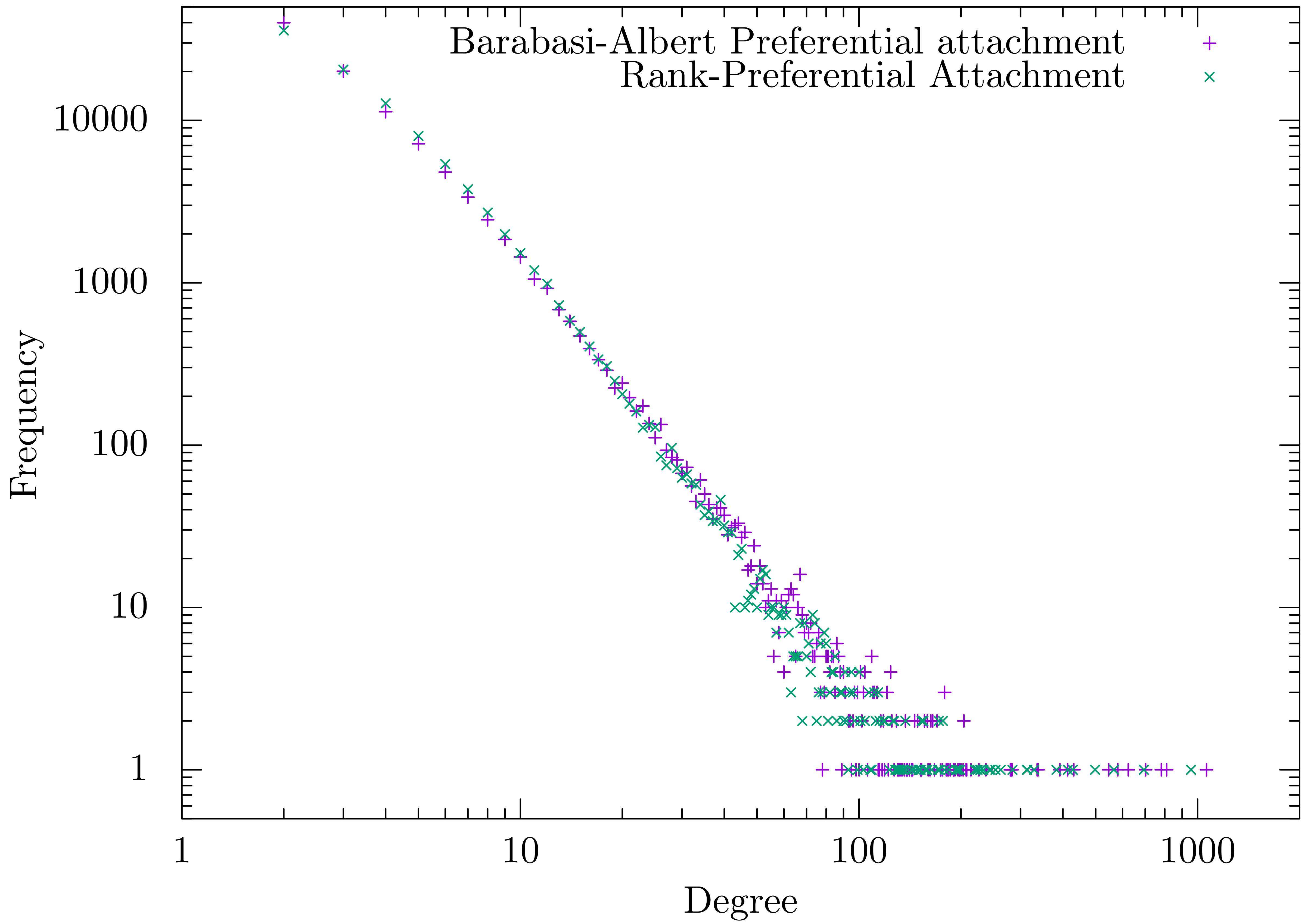
Function of node i's degree, e.g. BA model

$$p_i \propto f(\eta_i)$$

Function of some other intrinsic node property

Problem: How do we quantify how good a fit a model is to real data?

Moving away from descriptive statistics



Barabási Albert model

$$p_i \propto k_i$$

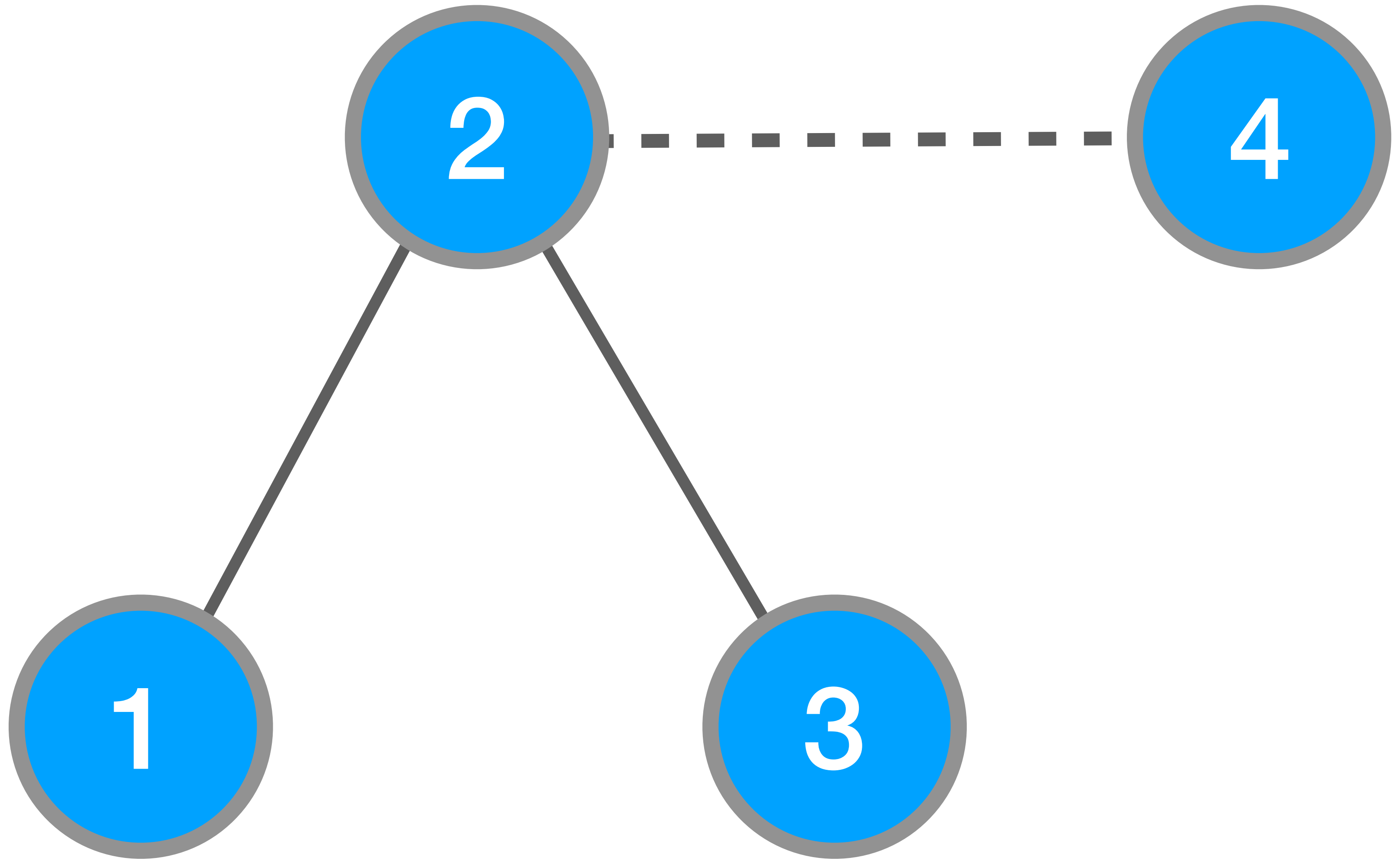
Static rank-preference model

$$p_i \propto i^{-\alpha}$$

Problem: different models may generate networks with similar descriptive statistics!

Different approach: Model likelihood

Likelihood of model given observation = probability of seeing observation given model



Likelihood of random/uniform model given by:

$$\mathbb{P}_{rand}(\textbf{Choose node } 2) = \frac{1}{3}$$

Likelihood of BA preferential attachment model given by:

$$\mathbb{P}_{BA}(\textbf{Choose node } 2) = \frac{2}{1+2+1} = \frac{1}{2}$$

[R. Clegg, B. Parker, M. Rio 2016: Likelihood-based assessment of dynamic network models]

Likelihood of model given observed period of network's evolution

Network (random variable): $G := G_t$

Observations (snapshots): $\mathbf{g} = \left(g_1, g_2, \ldots, g_n\right)$

Model possibly with parameters: $M(\theta)$

$$L(M(\theta)\,|\,\mathbf{g}) = \prod_{t=1}^{n} \mathbb{P}(G_t = g_t\,|\,G_{t-1} = g_{t-1}, \ldots, G_1 = g_1, M(\theta))$$

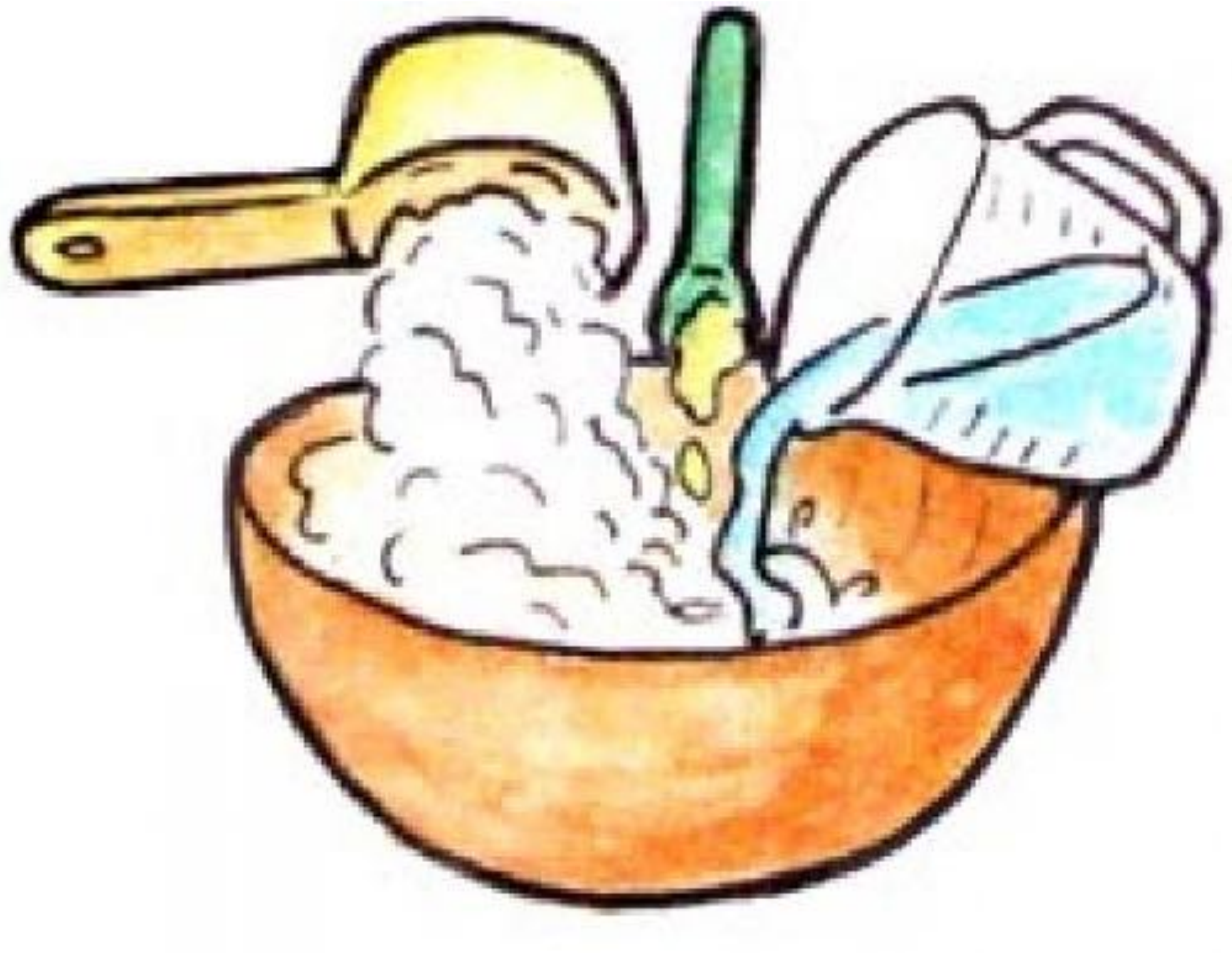
[R. Clegg, B. Parker, M. Rio 2016: Likelihood-based assessment of dynamic network models]

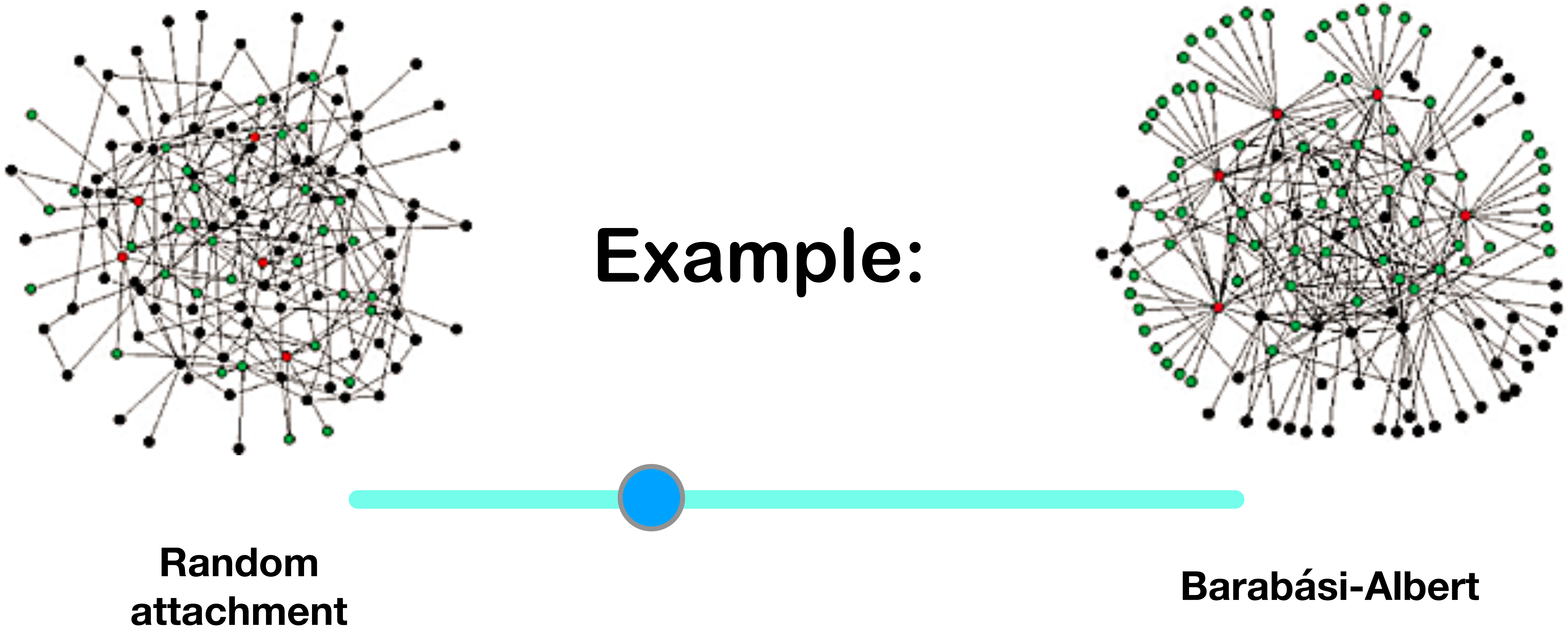
Likelihood: Remarks

- Quickly calculated, compared to generating networks
- Given a number of models, can define the 'best' as that which has the highest likelihood
- For models with parameters, can find maximum likelihood estimators for params

Assumption relaxation 1: Combining attachment probabilities

Idea: attachment in networks likely to be driven by a mixture of factors





This example explored in [Ghoshal, Chi, Barabási 2014: Uncovering the role of elementary processes in network evolution]

More formally:

$$p_i = \sum_l \beta_l p_i^{(l)} \quad \text{with} \quad \sum_l \beta_l = 1$$

a weighted combination of models

$p_i^{(l)}$ probability of choosing node i according to model l

[R. Clegg, B. Parker, M. Rio 2016: Likelihood-based assessment of dynamic network models]

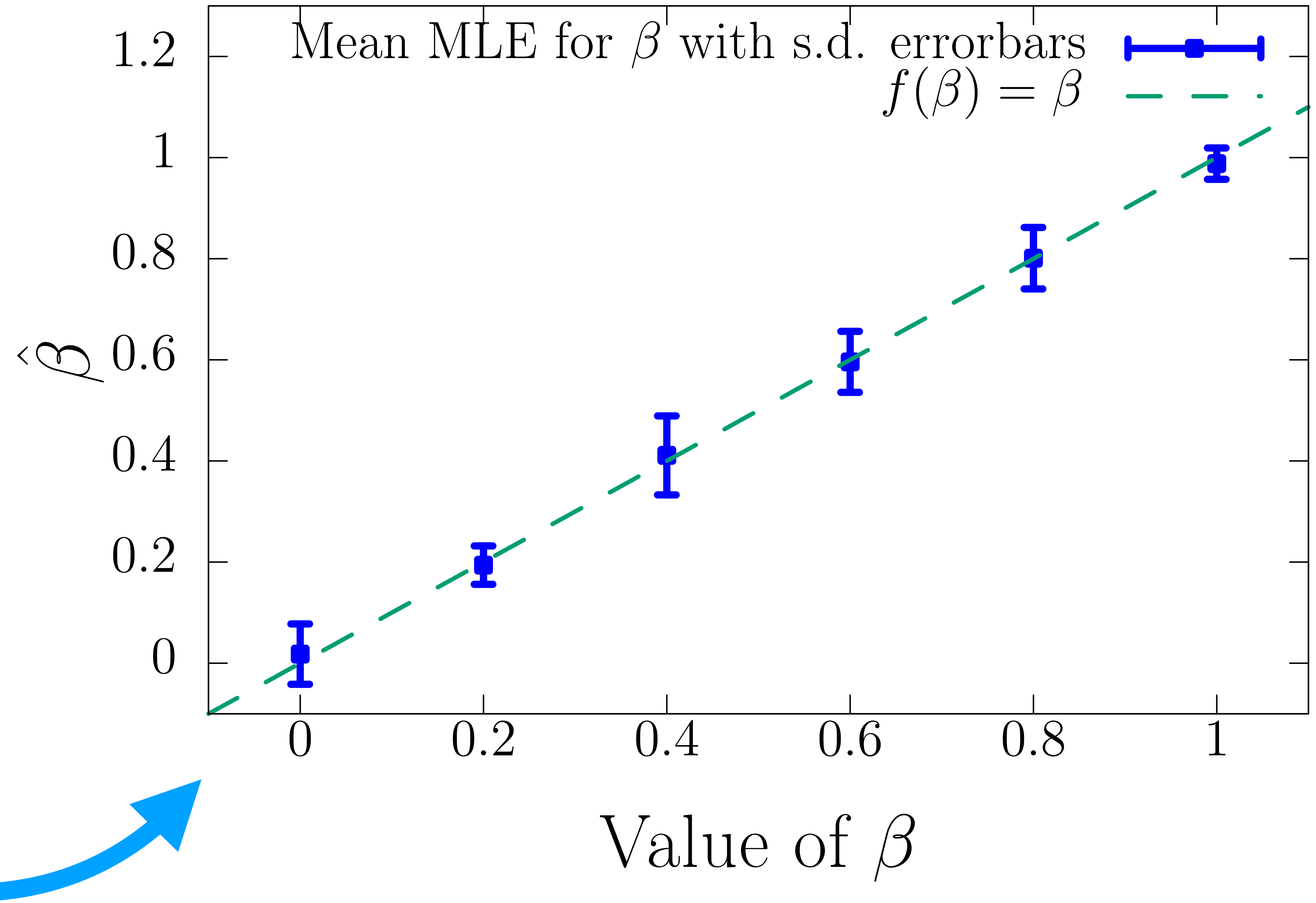
Can address the earlier problem with descriptive statistics using likelihood measure!

Distinguishing using maximum likelihood estimation

Having generated artificial networks using:

$$p_i = \beta p_i^{RP} + (1-\beta) p_i^{BA}$$

We can accurately recover the proportion β **as an MLE!**

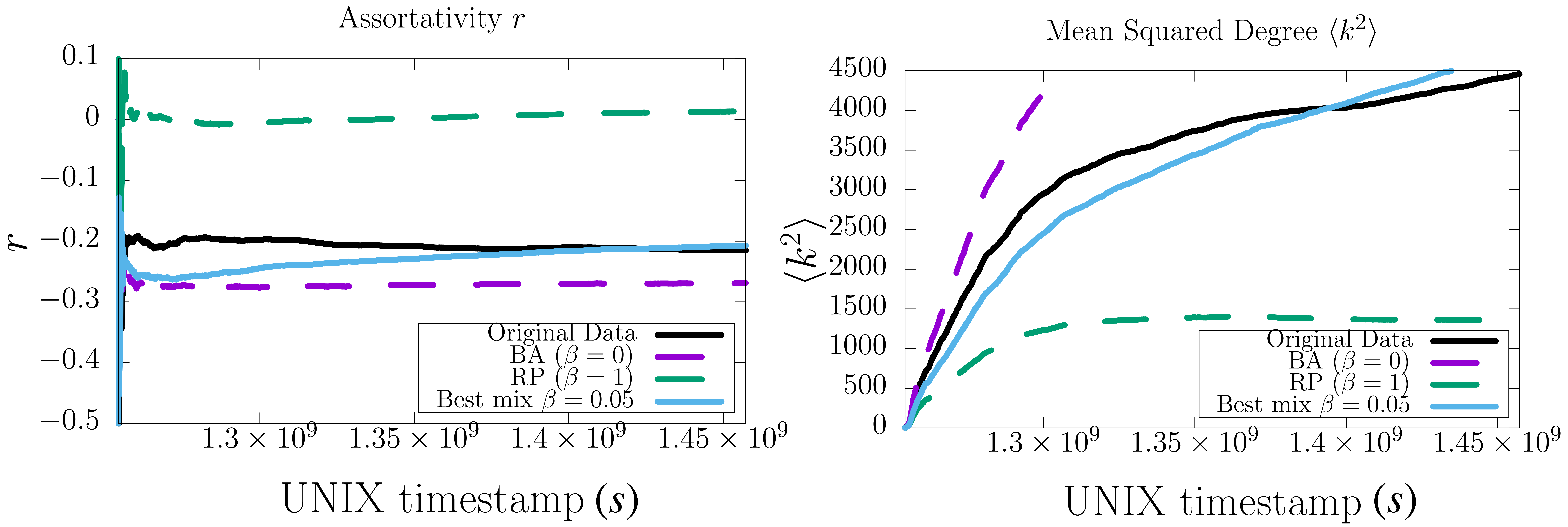


Real data example: Math Overflow Social Network

- Q&A site for mathematicians with problems
- Nodes are users
- An undirected edge between node A and B if A answers a question by B, A comments on B’s answer or question
- Multiple edges collapsed
- Models components tested: BA, static rank preference

Dataset from [A. Paranjape, A. Benson, J. Leskovec 2017: Motifs in temporal networks]

Real data example: Math Overflow Social Network



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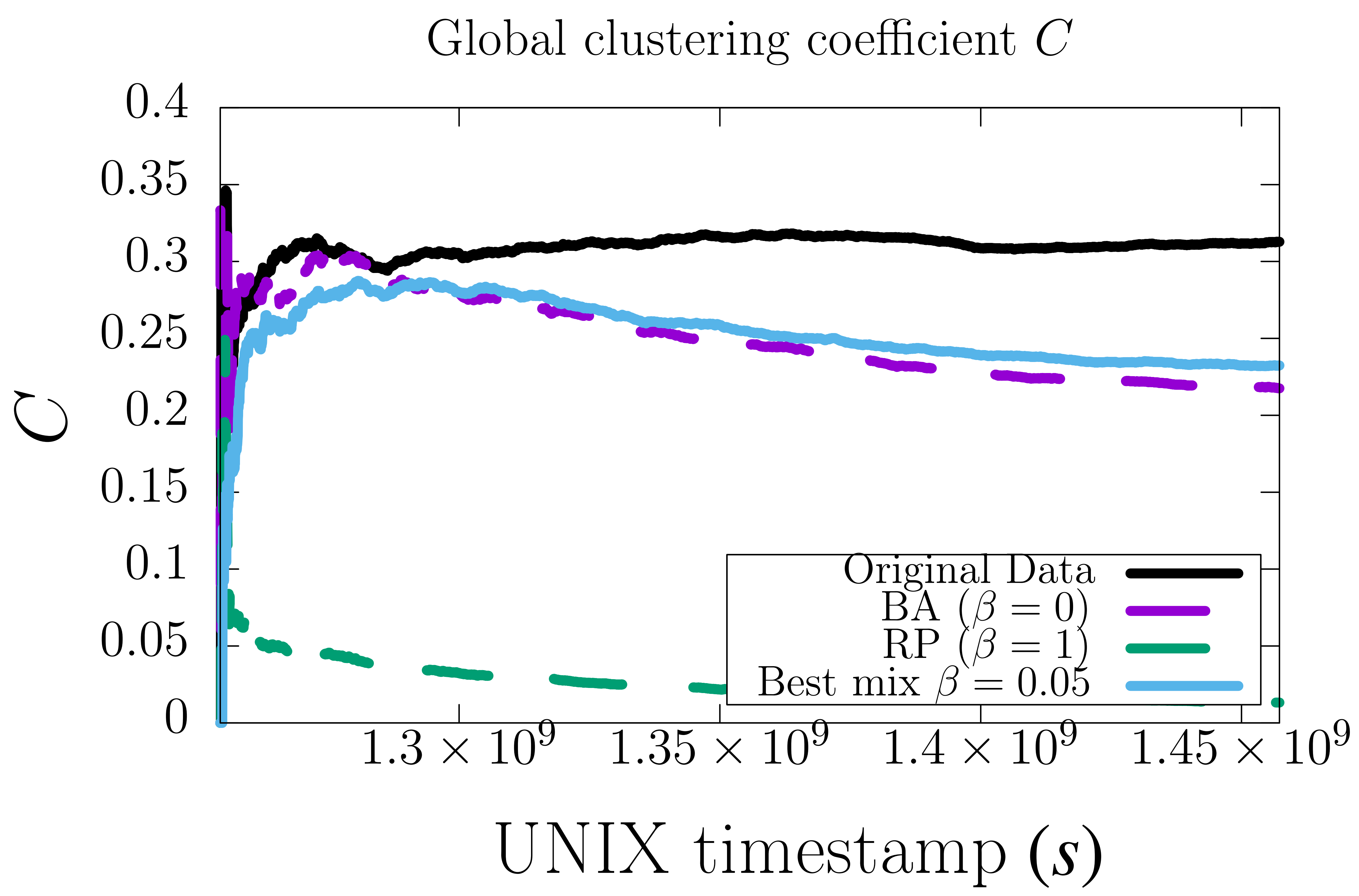
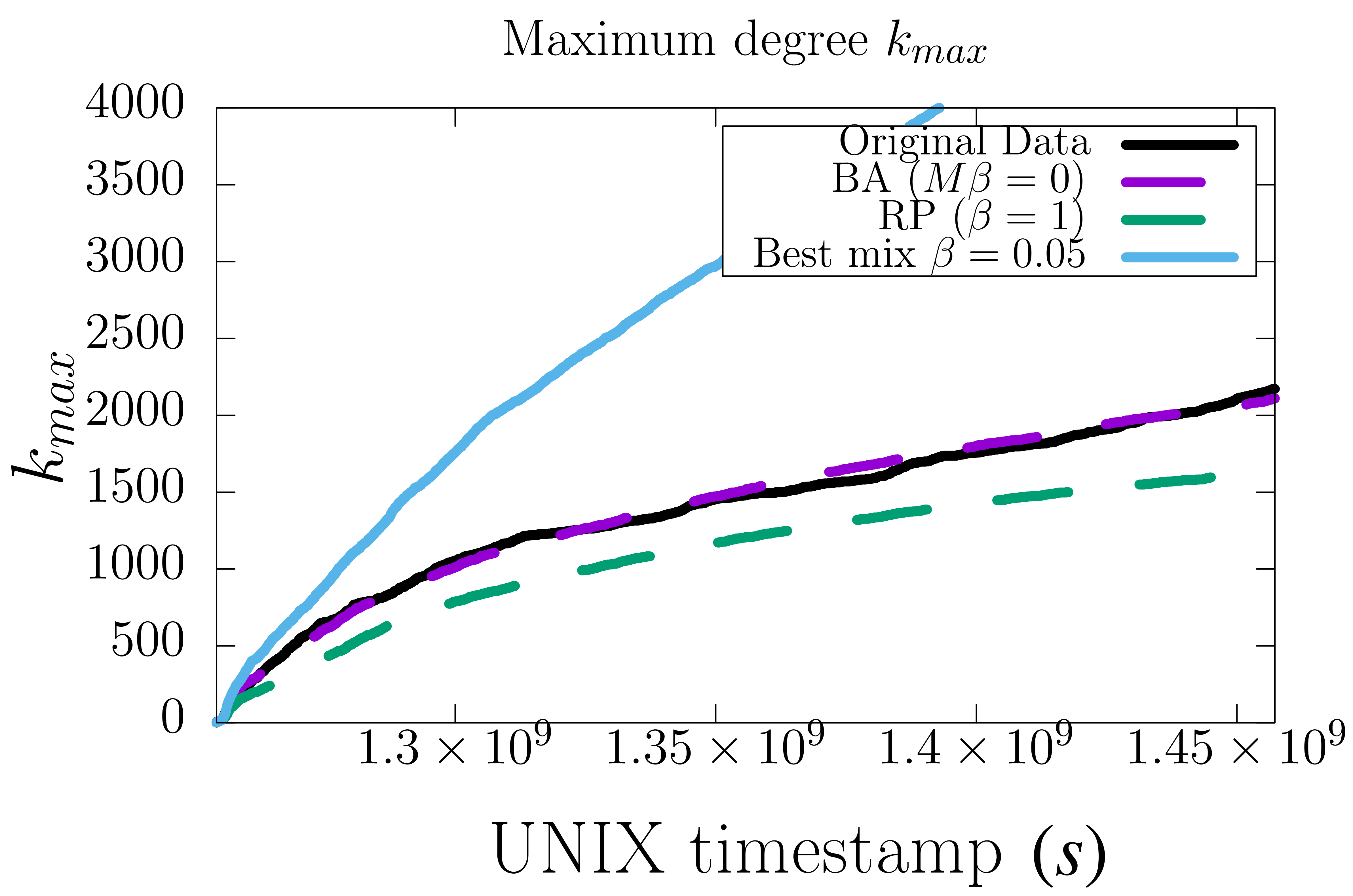


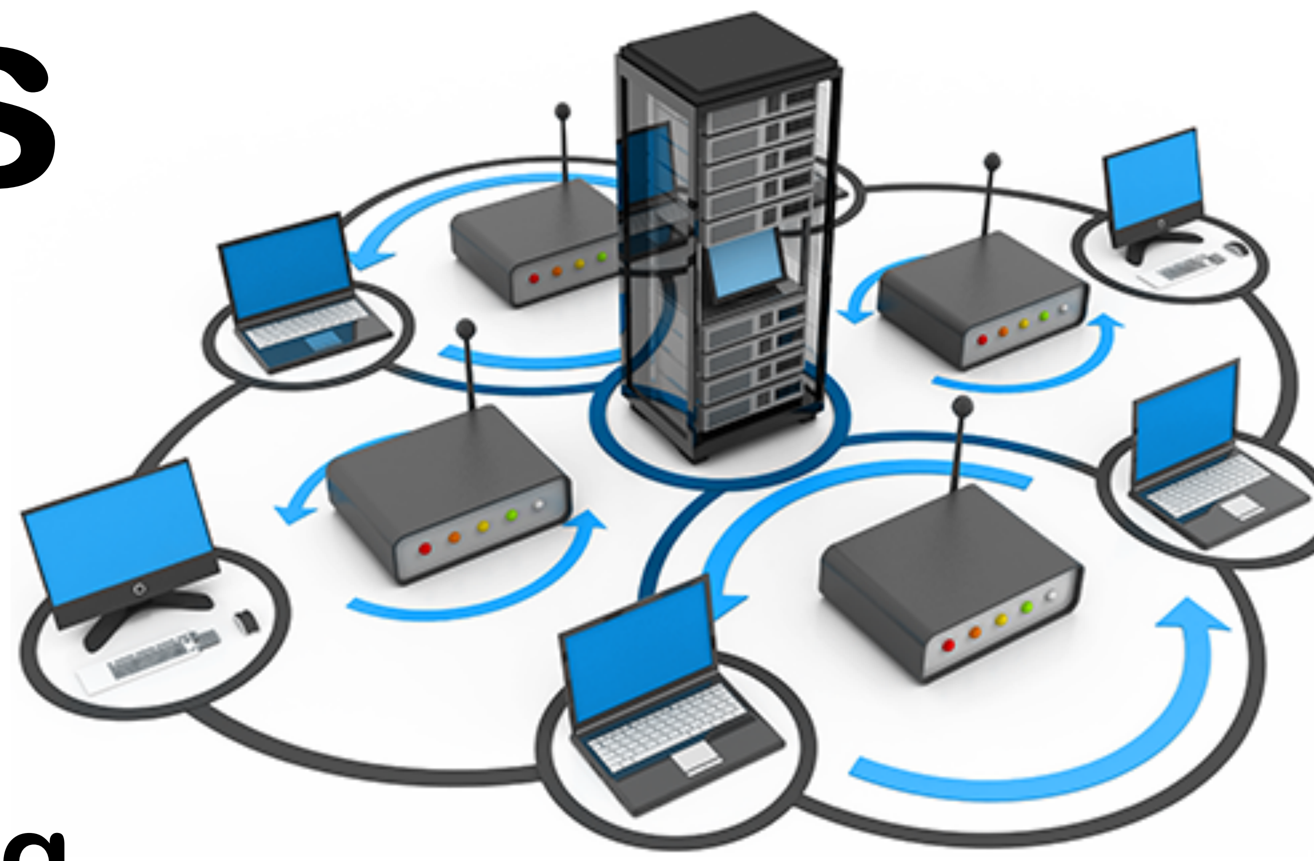


Dataset from [A. Paranjape, A. Benson, J. Leskovec 2017: Motifs in temporal networks]

Remarks

- Mixed model often generates network with better match on stats than single model components alone
- Process only gives the highest likelihood mix of the components tested - not guaranteed to be a good model.
- Searching through parameter space becomes expensive with more than two model components. Candidate problem for ML techniques.
- Work in progress: applying simulated annealing to model fitting

Assumption relaxation 2: Time varying models



In case you missed it

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- Networks may have changing growth regimes over time
- Can these changes be reflected in our modelling of networks?

Time varying model

For a set of models M_j which, given a network topology, assign probabilities to each node, introduce time varying model:

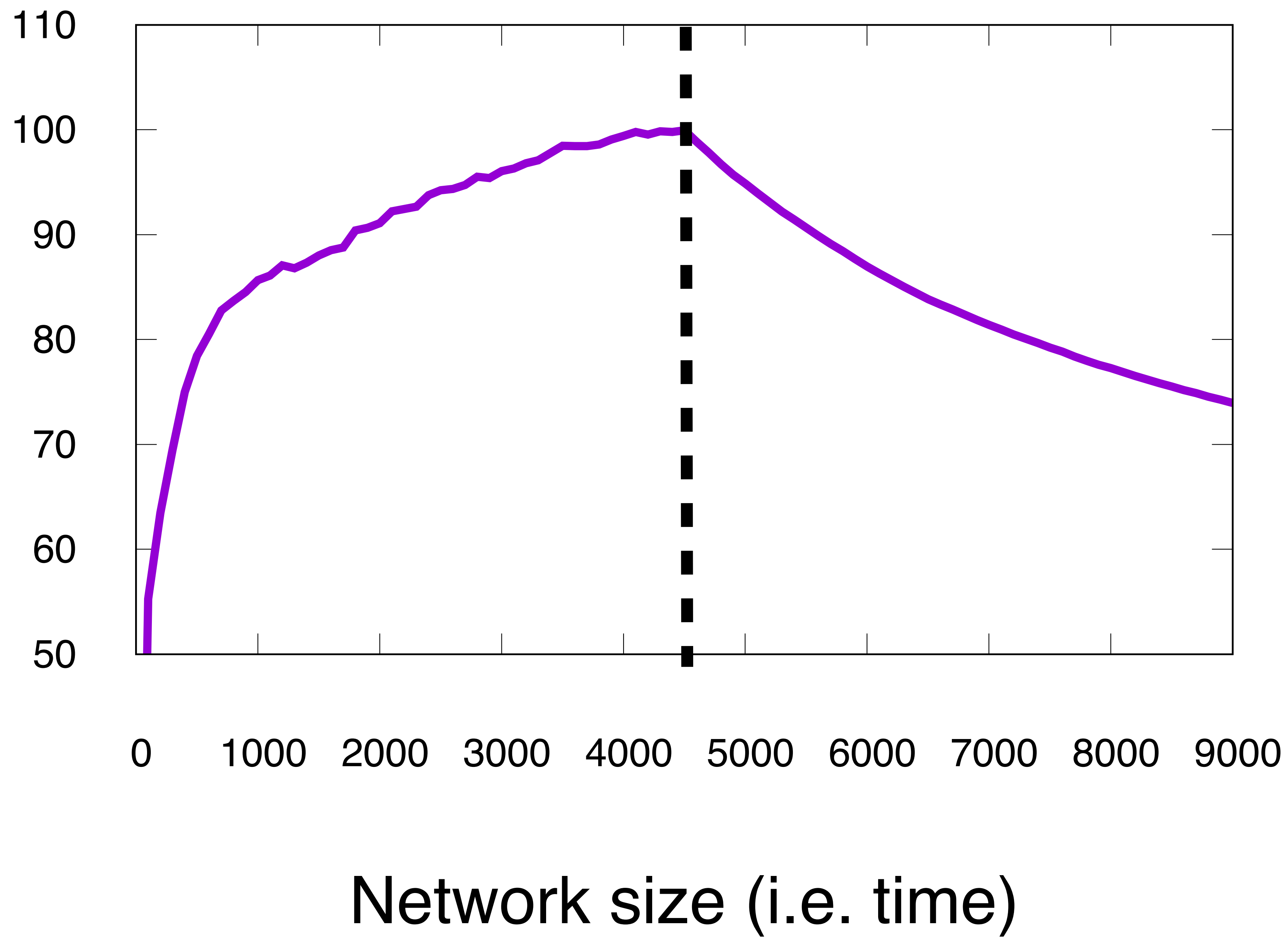
$$M(t) = \begin{cases} M_0 & t \in [t_0, t_1) \\ M_1 & t \in [t_1, t_2) \\ \vdots & \vdots \\ M_k & t \in [t_k, t_{k+1}) \end{cases}$$

Are such changes in a network's growth reflected in measurements we take of it?

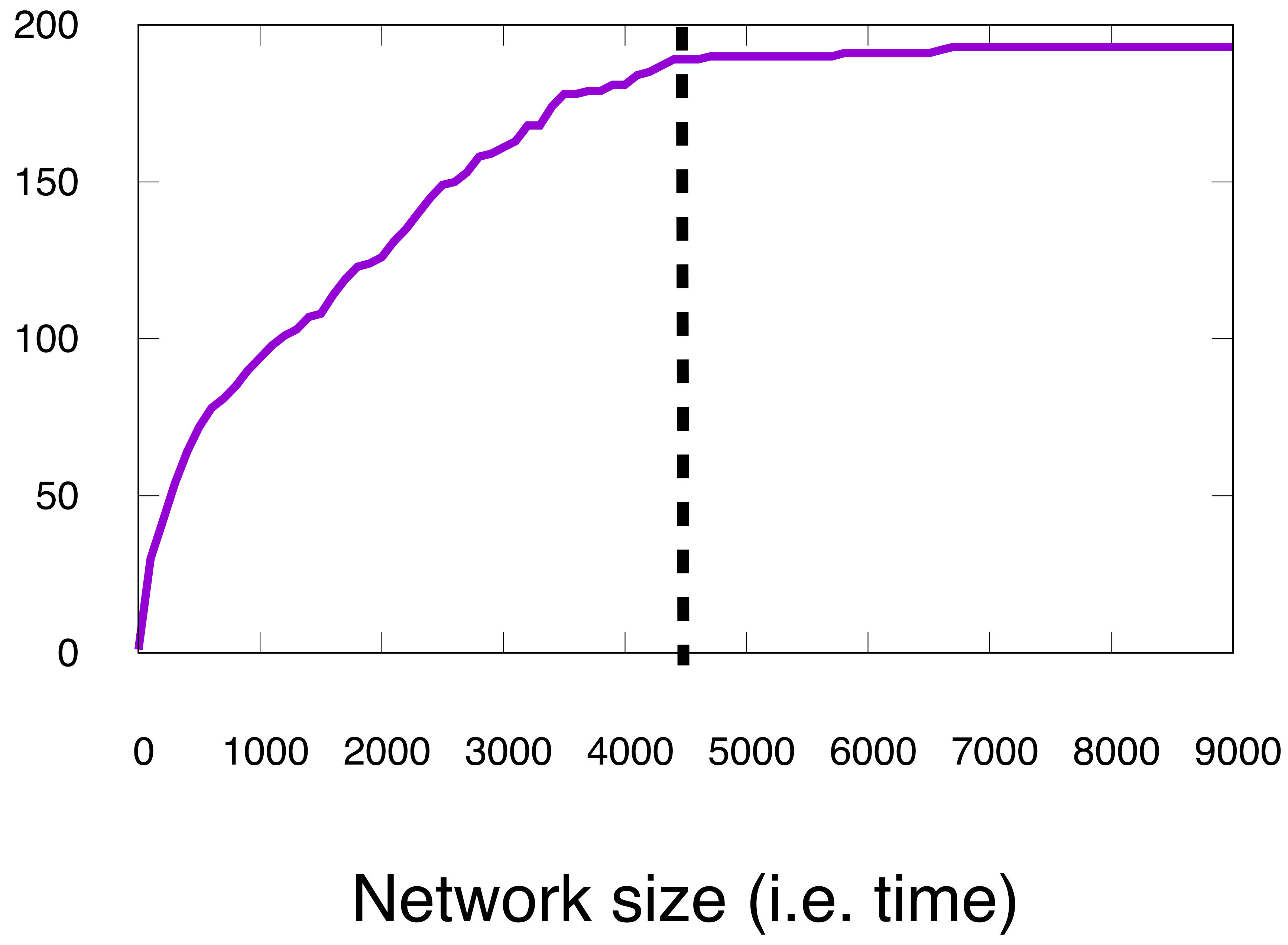
Sometimes yes

(Network generated with first half BA, second half neutral probabilities)

Mean squared degree $\langle k^2 \rangle$



Maximum degree k_{max}

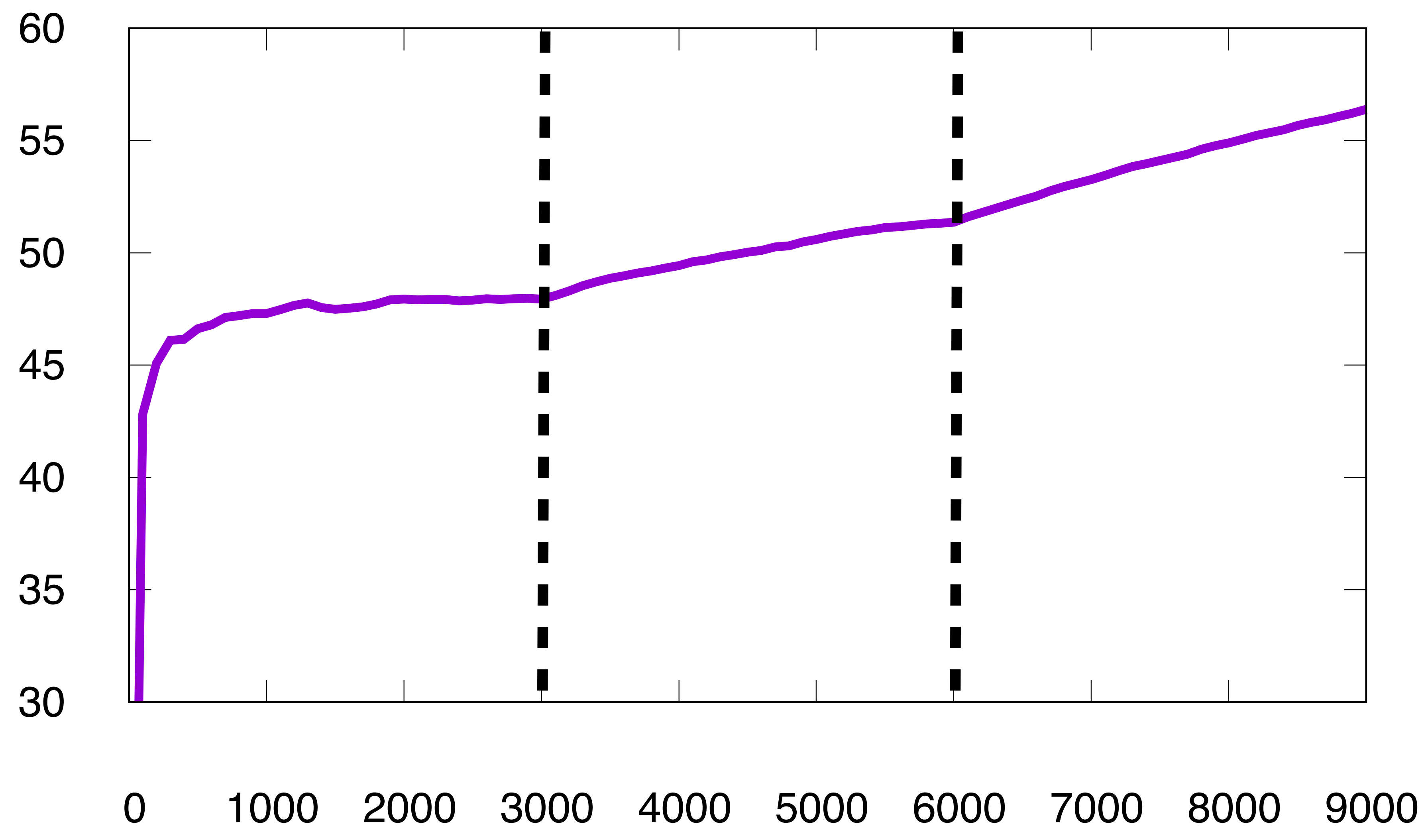


Are such changes in a network's growth reflected in measurements we take of it?

Other times less so...

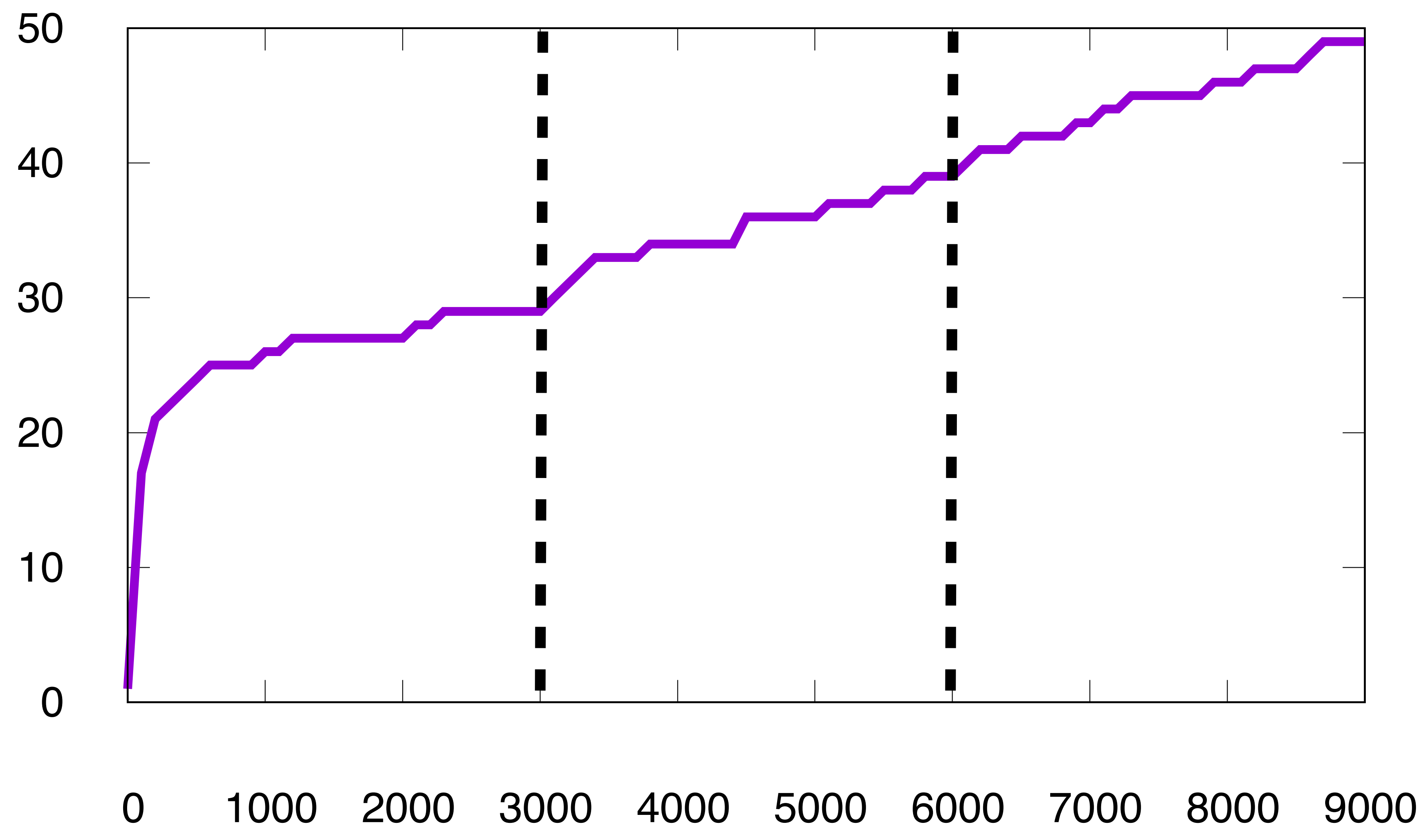
(Network first third: random, second: half random half BA, third: BA)

Mean squared degree $\langle k^2 \rangle$



Network size (i.e. time)

Maximum degree k_{max}

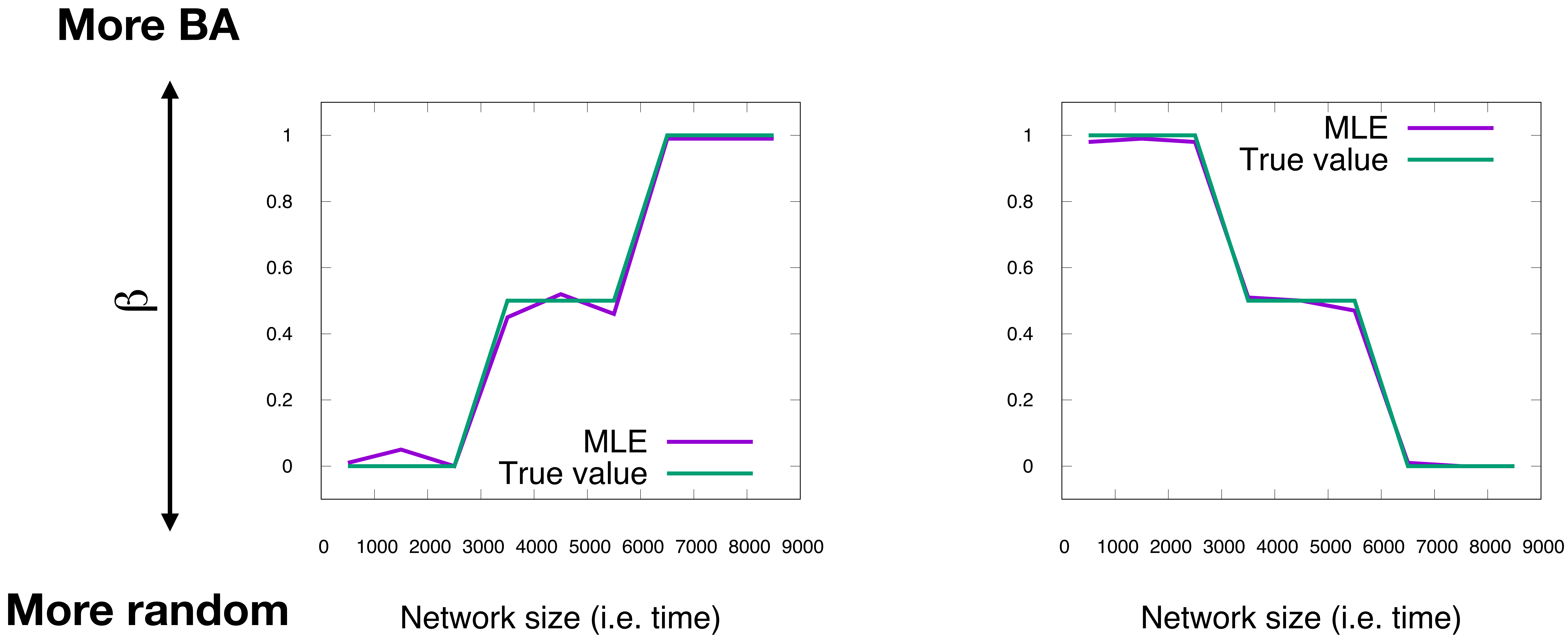


Network size (i.e. time)

Investigation with artificially generated networks

- Generate networks with different growth regimes
- What does this look like when we look at descriptive statistics?
- Can we detect these changes using likelihood-based techniques?

Tests on generated networks



Conclusions and future directions

- Mixed models allow us to uncover the different attachment mechanisms at play in a network's growth
- Fitting to different time periods may reveal how these mechanisms can change over time
- Currently developing a likelihood measure aimed at detection of changes in a network's growth regime
- Working on use with real data

Thank you for listening!
What are your questions?

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