Quantifying Link Stability in Mobile Ad Hoc Wireless Networks Using a Hidden Markov Model

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Overview

Problem

- In mobile ad hoc wireless networks, connections between nodes are established and broken intermittently due to node mobility and variations in the propagation channel, leading to **dynamically changing network topology**.

Questions

- How "complex" is the network? How many bits are needed to encode its random topology? What's the impact of link dynamics on the network's performance?

Our Solution

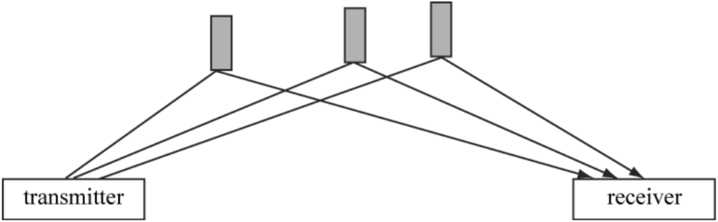
- Use **graph theory** and the information theoretic notion of **entropy rate** to measure the **topological uncertainty of the wireless network** and to quantify how quickly the underlying topology is varying with time.

Topological Uncertainty in Wireless Networks

- Justin P. Coon, "Topological Uncertainty in Wireless Networks" in IEEE Global Communications Conference (GLOBECOM), 2016.
- Justin P. Coon, and P. J. Smith "Topological Entropy in Wireless Networks Subject to Composite Fading" in IEEE International Conference on Communications (ICC), 2017.
- A. Cika, J. P. Coon, and S. Kim, "Effects of Directivity on Wireless Network Complexity", in IEEE International Symposium Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), 2017.
- M.-A. Badiu, and J. P. Coon, "On the Distribution of Random Geometric Graphs" in IEEE International Symposium on Information Theory (ISIT), 2018.
- J. P. Coon, C. P. Dettmann, and O. Georgiou, "Entropy of spatial network ensembles" in Physical Review E, 2018.
- **A. Cika, M.-A. Badiu, J. P. Coon, and S. E. Tajbakhsh, "Entropy rate of time-varying wireless networks", in IEEE Global Communications Conference (GLOBECOM), 2018.**
- J. P. Coon, M.-A. Badiu, and D.Gündüz, "On the conditional entropy of wireless networks", in IEEE International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), 2018.
- **A. Cika, M.-A. Badiu, and J. P. Coon, "Quantifying Link Stability in Ad Hoc Wireless Networks Subject to Ornstein-Uhlenbeck Mobility", in IEEE International Conference on Communications (ICC), 2019, *to appear*.**

Background: Rayleigh Fading

- Rayleigh fading arises when the signal arriving at the receiver has **undergone multiple reflections** with **NO** direct signal path between receiver and transmitter.
- The fading characteristics of the channel are determined by the **maximum Doppler frequency**, $\nu_{\max}$.



In a typical multipath propagation environment, the channel impulse response is $G_k \sim \mathcal{CN}\left(0, \lambda^2\right)$. The envelope of the channel response, $|G_k|$, will therefore be **Rayleigh distributed**.

Background: Signal to Noise Ratio (SNR)

In mobile ad hoc wireless networks, a transmission from node i to node j at any time step k is **successful** (the link is active) if the **SNR** of the link, $\Gamma_k^{i,j}$, is greater than a certain threshold γ_{th} determined by the communication hardware, and the modulation and coding scheme of the wireless system.

If we assume a single input single output link with additive Gaussian noise, then $\Gamma_k^{i,j}$ at any time step k is given by

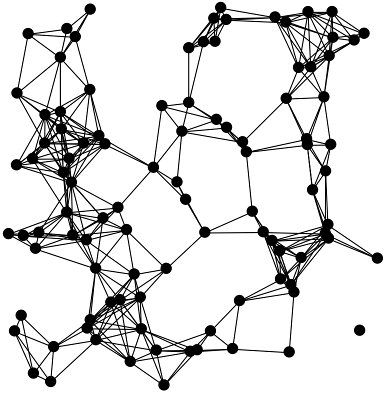
$$\Gamma_k^{i,j} = \psi \left(R_k^{i,j} \right)^{-\eta} |G_k^{i,j}|^2,$$

where $R_k^{i,j}$ is distance between nodes i and j at time step k, η is the path loss exponent (typically $\eta \geq 2$), and ψ is a constant depending on different parameters such as transmit power, antenna properties, and wavelength.

Part 1: Static Network subject to Rayleigh Fading

Random Geometric Graphs

- We model the time-varying wireless network as a time-ordered sequence of soft undirected random geometric graph (RGG).



Application examples:

- wireless networks
- social networks
- transportation networks

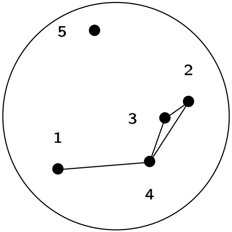
Properties of an ensemble:

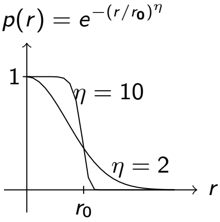
- connectedness
- degree distribution
- topological structure

System Model

- n wireless static nodes, located randomly in a space $\mathcal{K} \subset \mathbb{R}^2$.
- The locations of the nodes, $\mathbf{Z} = (Z^i)_{i \in \mathcal{V}_n}$, are **iud** in $\mathcal{K}$.
- Bounding geometries **circle/square/triangle**, and **Rayleigh fading**.
- $L_k^{i,j}$ conditioned on the pair distance $R^{i,j} = \|Z^i - Z^j\|$ is

$$L_k^{i,j} | R^{i,j} = \begin{cases} 0, & \text{if } 0 < \Gamma_k^{i,j} < \gamma_{th}, \\ 1, & \text{if } \gamma_{th} \le \Gamma_k^{i,j} < \infty. \end{cases}$$





Two-State Markov Model for Connection Links

Conditioned on the pair distances, we model the evolution of each edge (i,j) as a **stationary Markov chain** with on and off states.

$$
\begin{tikzcd}
0 \arrow[loop left, "P_{0,0}"] \arrow[r, bend left, "P_{0,1}"] & 1 \arrow[loop right, "P_{1,1}"] \arrow[l, bend left, "P_{1,0}"]
\end{tikzcd}
$$

- $\mathbb{P}\left(L_k^{i,j} = 1 | R^{i,j} = r^{i,j}\right) = \mathrm{e}^{-\left(r^{i,j}/r_0\right)^{\eta}}, \quad r_0 \sim \left(1/\gamma_{th}\right)^{1/\eta}.$
- In a communication system with transmission rate of B [symbols/s]

$$\mathbb{P}\left(L_k^{i,j} = 1 - a | L_{k-1}^{i,j} = a, R^{i,j} = r^{i,j}\right) \approx \frac{\mathrm{LCR}(r^{i,j})}{P\left(L_{k-1}^{i,j} = a | R^{i,j}\right) \times B}.$$

Entropy Rate

- The entropy rate of the stationary stochastic process $\mathbf{L}_k = (L_k^{i,j})_{i<j}$ is defined by
$$H(\mathcal{L}) = \lim_{k \to \infty} \frac{1}{k} H(\mathbf{L}_1, \ldots, \mathbf{L}_k).$$

- It measures the average **minimum description length** of a stationary stochastic process capturing the state of the dynamic system.

- A high entropy rate indicates that the topology is frequently changing over time, leading to an **increase in overhead** throughout the network.

- It can represent an **accurate metric of link stability** in dynamic networks; it measures the uncertainty of the future state of the link given its current state.

Computational Issues

$$\mathbb{P}(\mathbf{L}_1, \dots, \mathbf{L}_k) = \int_{\mathcal{R}=[0,D]^{n(n-1)/2}} \mathbb{P}(\mathbf{L}_1, \dots, \mathbf{L}_k | \mathbf{R})\, f_{\mathbf{R}}(\mathbf{r}) d\mathbf{r}.$$

- After averaging, the resulting stochastic process $\mathbf{L}_k$ inherits the stationary property but **not** the Markov property.
- Evaluation of the integral requires the joint probability distribution function $f_{\mathbf{R}}(\mathbf{r})$ of pair distances; **expressions are not available for $\mathbf{n > 2}$** when nodes are confined inside a triangle/square. For $n = 3$ expressions are available if nodes are confined in a circle [1].
- We resort to bounding the entropy rate of a random geometric graph.

[1] M.-A. Badiu, and J. P. Coon "On the Distribution of Random Geometric Graphs" in IEEE International Symposium on Information Theory (ISIT), 2018.

Bounds on the Entropy Rate

Upper Bound on the Entropy Rate

$$H\left(\mathbf{L}_1, \ldots, \mathbf{L}_k\right) \leq \sum_{i<j} \left[(k-1) H\left(L_2^{i,j} | L_1^{i,j}\right) + H\left(L_1^{i,j}\right)\right].$$

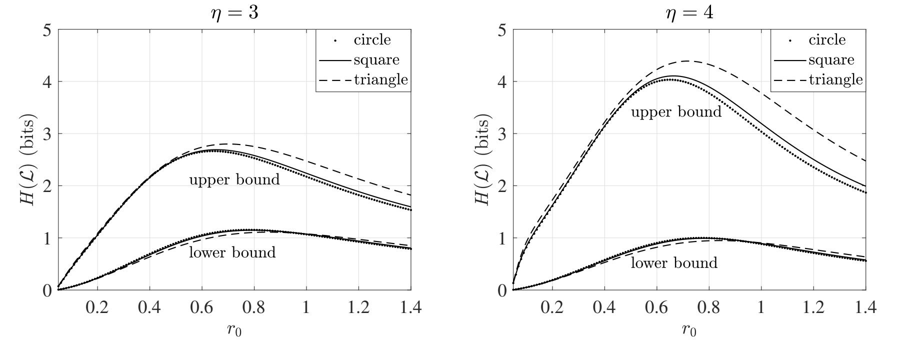
Lower Bound on the Entropy Rate

$$H\left(\mathbf{L}_1, \mathbf{L}_2, \ldots, \mathbf{L}_t\right) \geq \sum_{i<j} \left[(t-1) H\left(L_2^{i,j} | L_1^{i,j}, R^{i,j}\right) + H\left(L_1^{i,j} | R^{i,j}\right)\right].$$

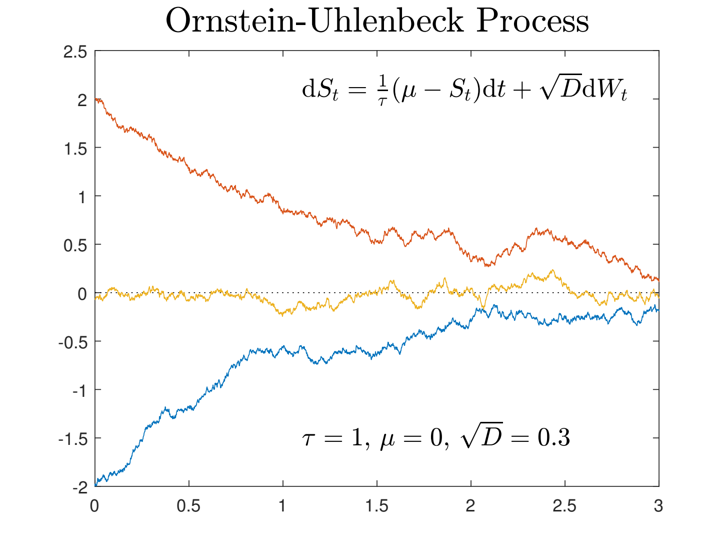
$$\binom{n}{2} H\left(L_2^{1,2} | L_1^{1,2}, R^{1,2}\right) \leq H(\mathcal{L}) \leq \binom{n}{2} H\left(L_2^{1,2} | L_1^{1,2}\right).$$

Numerical Results

- We consider a communication system using 802.11a/g protocols with symbol rate $B = 12$ MBd.
- Evaluate the entropy rate of a fifty-node RGG; bounding geometries: square of unit side length, circle of radius $1/\sqrt{\pi}$, and equilateral triangle of side length $2/\sqrt[4]{3}$; maximum Doppler frequency $\nu_{\max} = 500$ Hz.



Mobility Model



System Model

- Consider 2 arbitrary nodes (mobile wireless devices) **moving randomly** over a two-dimensional plane.
- The locations of the nodes at time $t \geq 0$ are given by $\left(X_t^1, Y_t^1\right)$ and $\left(X_t^2, Y_t^2\right)$.
- The separation distance between nodes at time t is

$$R_t = \sqrt{\left(X_t^2 - X_t^1\right)^2 + \left(Y_t^2 - Y_t^1\right)^2}.$$

- We assume there is no fading affecting the link between nodes, i.e., $|G_t|^2 = 1$. Then, Γ_t at any time t is given by

$$\Gamma_t = \psi R_t^{-\eta}.$$

Markov Model of Link Process

Instead of observing the locations of the mobile nodes continuously, we monitor them at regular time steps $k = t_0 + k\Delta t$, $k \in \mathbb{N}$ and $\Delta t > 0$.

The random variable L_k denotes the link state between nodes at any time step k, where 1(0) defines whether the link exists (does not exist)

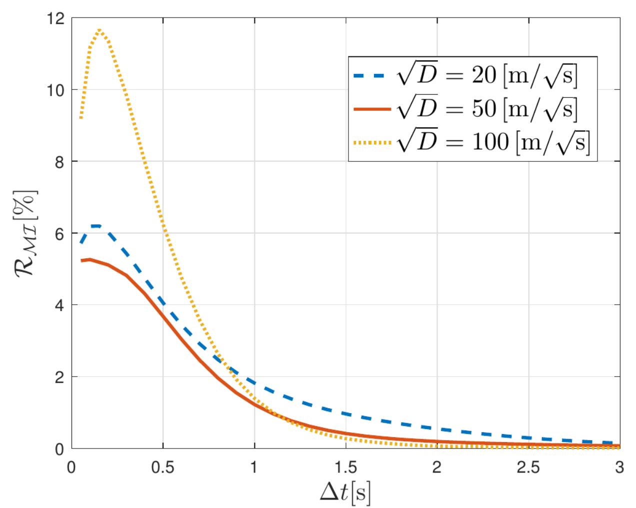
$$L_k = \begin{cases} 1, & \text{if } R_k \leq r_0, \\ 0, & \text{otherwise}, \end{cases}$$

where $r_0 = (\psi/\gamma_{th})^{\frac{1}{\eta}}$ is the typical connection range.

To assess the validity of the first-order Markov assumption we evaluate a mutual-information-based metric as a function of the sampling interval Δt

$$\mathcal{R}_{\mathcal{MI}} = \frac{I\left(L_k; L_{k-2} | L_{k-1}\right)}{I\left(L_k; L_{k-1}, L_{k-2}\right)}.$$

Numerical evaluation for the mutual information ratio $\mathcal{R}_{\mathcal{MI}}$ versus the sampling interval Δt; mobility parameters: $\tau = 1\mathrm{s}$, $\sqrt{D} = 100\mathrm{m}/\sqrt{\mathrm{s}}$, $\beta = 10\mathrm{m}$, and connection range $r_0 = 50\mathrm{m}$.



Entropy Rate as a Link Stability Metric

The link state evolution $\{L_k, k \in \mathbb{N}\}$ is modeled as a stationary Markov chain, and its entropy rate is equal to the transition entropy

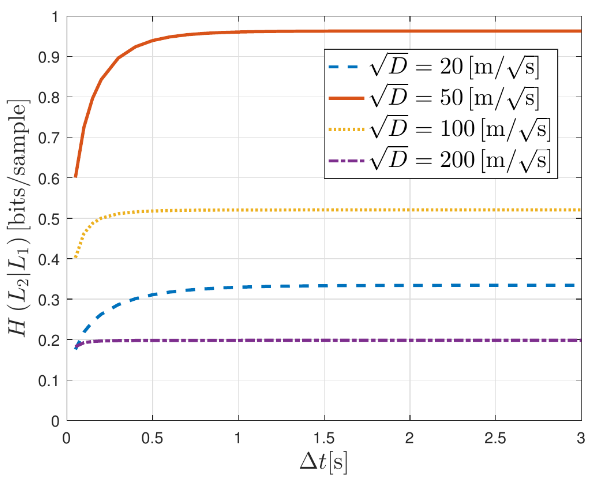
$$H(L_2|L_1) = -\sum_{b\in\{0,1\}} \mathbb{P}(L_1 = b) \times \sum_{a\in\{0,1\}} \mathbb{P}(L_2 = a|L_1 = b) \log_2 \mathbb{P}(L_2 = a|L_1 = b).$$

$$\mathbb{P}(L_2 = a|L_1 = b) = \frac{\int_{r_3\in\mathbb{R}^+}\int_{r_2\in\mathcal{I}_a}\int_{r_1\in\mathcal{I}_b} f_{R_1,R_2,R3}(r_1, r_2, r_3)\,\mathrm{d}r_1\mathrm{d}r_2\mathrm{d}r_3}{\int_{r\in\mathcal{I}_b} f_R(r)\mathrm{d}r}.$$

where the state variables $a, b \in \{0, 1\}$ determine the integration intervals $\mathcal{I}_a$ and $\mathcal{I}_b$, respectively.

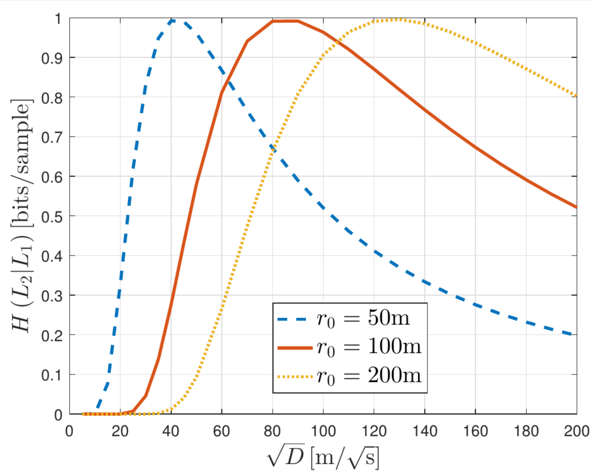
Numerical Results (I)

Numerical evaluation for the entropy rate $H\left(L_2|L_1\right)$ versus the sampling interval Δt; mobility parameters: $\tau = 1\mathrm{s}$, $\beta = 10\mathrm{m}$, $r_0 = 50\mathrm{m}$.



Numerical Results (II)

Numerical evaluation for the entropy rate $H\left(L_2|L_1\right)$ versus the diffusion coefficient $\sqrt{D}$; mobility parameters: $\tau = 1\mathrm{s}$, $\Delta t = 1\mathrm{s}$, and $\beta = 10\mathrm{m}$.



Part 3: Mobile Network subject to Rayleigh Fading

Markov Model of Distance Process

The squared separation distance between nodes 1 and 2 at any time t is given by

$$P_t = \left(X_t^2 - X_t^1\right)^2 + \left(Y_t^2 - Y_t^1\right)^2 .$$

The **Stochastic Differential Equation** of P_t is given by

$$\mathrm{d}P_t = \left(4D - \frac{2}{\tau}P_t\right)\mathrm{d}t + 2\sqrt{2D}\sqrt{P_t}\,\mathrm{d}W_t, \quad P_0 = p_0.$$

The square distance process $(P_t, t \geq 0)$ is a **stationary ergodic Bessel process**.

Markov Model of Distance Process

The **probability density of the squared distance** at time t, conditioned on its value at the current time s, is

$$f_{P_t|P_s}(p_t|p_s) = \chi^2\left[2p_t c; 2, 2p_s u\right],$$

with **2** degrees of freedom and parameter of non-centrality $\mathbf{2p_s u}$, where

$c = \frac{1}{2D\tau(1-\exp[-k(t-s)])}$, and $u = c\exp\left[-k(t-s)\right]$.

The **steady state density function** is

$$f_P(p) = \frac{1}{2D\tau}\exp\left[-\frac{p}{2D\tau}\right].$$

Hidden Markov Model of Link Process (I)

Time Discretization

- Instead of observing the locations of the mobile nodes continuously, we monitor them at regular time steps $k = t_0 + k\Delta t$, $k \in \mathbb{N}$ and $\Delta t > 0$.
- The normalized autocovariance function of the squared envelope $\{|G_t|^2, t \geq 0\}$ is

$$\frac{\mathrm{E}\left\{|G_t|^2|G_{t+\Delta t}|^2\right\} - \mathrm{E}^2\left\{|G_t|^2\right\}}{\mathrm{E}^2\left\{|G_t|^2\right\}} = \mathrm{J}_0^2\left(2\pi\nu_{\max}\Delta t\right),$$

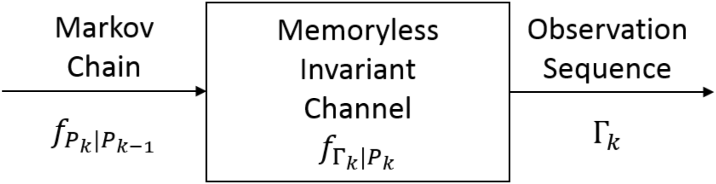
- In order to guarantee independence between two consecutive channel gain samples, Δt must satisfy the following condition

$$\Delta t \geq \frac{0.18}{\nu_{\max}}.$$

Hidden Markov Model of Link Process (II)

- The SNR at any time step k is $\Gamma_k = \psi P_k^{-\eta/2} |G_k|^2$.
- The channel gain is exponentially distributed with mean λ^2, i.e. $|G_k|^2 \sim \text{Exp}\left(\lambda^2\right)$ for any time step k. Then, the **observation conditional density** is

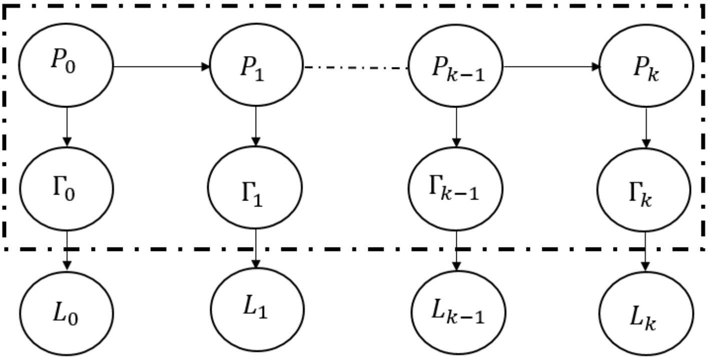
$$f_{\Gamma_k|P_k}\left(\gamma_k|p_k\right) = \frac{p_k^{\eta/2}}{\psi\lambda^2} \exp\left[-\gamma_k \frac{p_k^{\eta/2}}{\psi\lambda^2}\right].$$



Hidden Markov Model of Link Process (III)

L_t denotes the link state between nodes at any time t

$$\mathbb{P}\left(L_k = 1 | \Gamma_k = \gamma_k\right) = \begin{cases} 0, & \text{if } 0 < \gamma_k < \gamma_{th}, \\ 1, & \text{if } \gamma_{th} \leq \gamma_k < \infty. \end{cases}$$



Conclusions & Future Work

The evolution of the link between any two nodes is modeled as a **Hidden Markov Process**, which can effectively predict the presence (or absence) of a connection according to its SNR.

- This model can be used to formulate a stability metric using the entropy rate, taking full advantage of the **correlation** between the link current and future state.
- Extension to $n > 2$ nodes.
- Design **link state prediction-based routing algorithms** using the hidden Markov model and the entropy rate as stability metric to choose the most stable route between nodes.

Thank you !

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Extra Slides

Measuring Complexity through Encoding

- The Kolmolgorov complexity of an object is defined as the smallest possible description of that object using a fixed, universal description language.
- Minimum Description Length (MDL) is the principle that the best encoding of a dataset is the one that compresses it the most.

$$\text{Complexity} \rightarrow \text{MDL} \rightarrow \text{Entropy}$$

Level Crossing Rate

It quantifies how often the signal level crosses the threshold γ_{th}, usually in the positive-going direction, and is defined as

$$\mathrm{LCR}\left(\gamma_{th}\right) = \sqrt{2\pi}\left(\frac{\gamma_{th}}{\gamma_0}\right)^{1/2} \nu_{i,j}\ \mathrm{e}^{-\gamma_{th}/\gamma_0}. \tag{1}$$

Expressing the level crossing rate as a function of pair distance $R_{i,j}$, we obtain

$$\mathrm{LCR}\left(r_{i,j}\right) = \sqrt{2\pi}\left(\frac{r_{i,j}}{r_0}\right)^{\eta/2} \nu_{i,j}\ \mathrm{e}^{-\left(r_{i,j}/r_0\right)^{\eta}}. \tag{2}$$

Upper Bound on Entropy Rate

- We derive an upper bound on the entropy rate

$$H\left(\mathbf{X}^1, \ldots, \mathbf{X}^t\right) \leq \sum_{i<j} \left[(t-1) H\left(X_{i,j}^2 | X_{i,j}^1\right) + H\left(X_{i,j}^1\right)\right] \tag{3}$$

- Dividing by t and taking the limit $t \to \infty$, we arrive at the entropy rate relation

$$H\left(\mathcal{X}\right) \leq \sum_{i<j} H\left(X_{i,j}^2 | X_{i,j}^1\right) \tag{4}$$

where

$$H\left(X_{i,j}^2 | X_{i,j}^1\right) = - \sum_{a \in \{0,1\}} P\left(X_{i,j}^1 = a\right) \times \sum_{b \in \{0,1\}} P\left(X_{i,j}^2 = b | X_{i,j}^1 = a\right) \log P\left(X_{i,j}^2 = b | X_{i,j}^1 = a\right) \tag{5}$$

Lower Bound on Entropy Rate

$$H\left(X_{1,2}^2|X_{1,2}^1, R_{1,2}\right) = -\int_0^D f_R(r_{1,2})\mathrm{d}r_{1,2}$$
$$\times \sum_{a\in\{0,1\}} \mathbb{P}\left(X_{1,2}^1 = a|R_{1,2} = r_{1,2}\right)$$
$$\times \sum_{b\in\{0,1\}} \left[\mathbb{P}\left(X_{1,2}^2 = b|X_{1,2}^1 = a, R_{1,2} = r_{1,2}\right)\right.$$
$$\left.\times \log \mathbb{P}\left(X_{1,2}^2 = b|X_{1,2}^1 = a, R_{1,2} = r_{1,2}\right)\right].$$

Bounds on the Entropy Rate

- $P(X_{i,j} = a)$ is the probability that edge (i, j) exists $(a = 1)$ or not $(a = 0)$, averaged over the pair distance $R_{i,j}$

$$P\left(X^1_{i,j} = a\right) = \int_0^D P\left(X^1_{i,j} = a \middle| R_{i,j} = r_{i,j}\right) f_{R_{i,j}}(r_{i,j})\, \mathrm{d}r_{i,j}$$

- In the same fashion

$$P\left(X^2_{i,j} = b \middle| X^1_{i,j} = a\right) = \int_0^D P\left(X^2_{i,j} = b \middle| X^1_{i,j} = a, R_{i,j} = r_{i,j}\right) f_{R_{i,j}}(r_{i,j})\, \mathrm{d}r_{i,j},$$

for each $a,\ b \in \{0, 1\}$.

Instantaneous Signal-to-Noise-Ratio (SNR)

$\gamma_{i,j}^{t}$ is the received SNR, and has an exponential distribution with probability density function

$$f(\gamma_{i,j}) = \frac{1}{\gamma_0} \mathrm{e}^{-\gamma_{i,j}/\gamma_0}, \quad \gamma_{i,j} \geq 0 \tag{6}$$

Distance Distribution

We can write the separation distance between nodes at time t as

$$R_t = \sqrt{\left(X_t^j - X_t^i\right)^2 + \left(Y_t^j - Y_t^i\right)^2}, \tag{7}$$

where $X_t^j - X_t^i \sim \mathcal{N}\left(\beta, D\tau\left(1 - \exp\left[-2t/\tau\right]\right)\right)$ and $Y_t^j - Y_t^i \sim \mathcal{N}\left(0, D\tau\left(1 - \exp\left[-2t/\tau\right]\right)\right)$ are independent random variables.

By a simple transformation of random variables, it is easy to show that $R_t \sim \mathrm{Rice}\left(\beta, \sqrt{g_t}\right)$ for all t, and its probability density function is given by

$$f_R(r;t) = \frac{r}{g_t} \exp\left[\frac{-\left(r^2+\beta^2\right)}{2g_t}\right] \mathrm{I}_0\left(\frac{\beta r}{g_t}\right), \tag{8}$$

with $g_t = D\tau\left(1 - \exp\left[-2t/\tau\right]\right)$ and I_0 being the modified Bessel function of the first kind with order zero.

The trivariate distribution of the Rician random variables R_1, R_2, and R_3

$$
\begin{aligned}
& f_{R_1,R_2,R_3}(r_1, r_2, r_3) = \\
& \frac{r_1 r_2 r_3}{|\circ|} \exp\left\{ -\frac{1}{2} \left(\sum_{i=1}^{3} w_{ii} r_i^2 + \beta^2 w_4 \right) \right\} \\
& \times \sum_{q=0}^{\infty} \sum_{p=-\infty}^{\infty} \varepsilon_k (-1)^{q+p} \mathrm{I}_q(w_3 \beta r_3) \mathrm{I}_q(w_{32} \beta r_2 r_3) \\
& \times \mathrm{I}_p(w_1 \beta r_1) \mathrm{I}_p(w_{12} r_1 r_2) \mathrm{I}_{q+p}(w_2 \beta r_2),
\end{aligned}
\tag{9}
$$

where $w_1 = w_{11} + w_{12}$, $w_2 = w_{22} + w_{23} + w_{12}$, $w_3 = w_{33} + w_{23}$, $w_4 = w_1 + w_2 + w_3$, I_n is the modified Bessel function of the first kind and order n, $|\circ|$ is the determinant of the covariance matrix, and ε_k is the Neumann factor $(\varepsilon_0 = 1, \varepsilon_n = 2 \text{ for } n = 1, 2, \ldots)$.