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Simplicial Models of Social Contagion

Iacopo Iacopini, Giovanni Petri, Alain Barrat, Vito Latora
School of Mathematical Sciences - Queen Mary University of London
Centre for Advanced Spatial Analysis - University College London

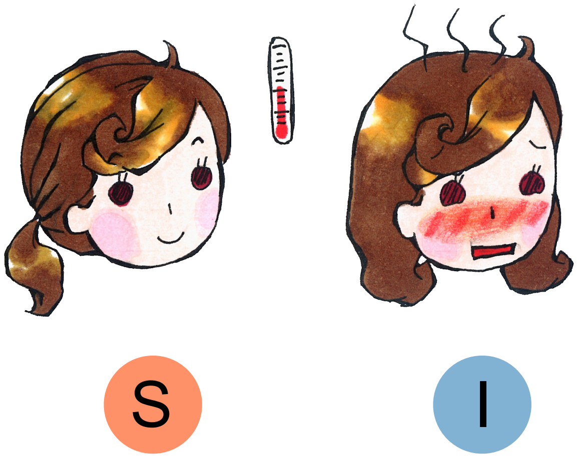
MoN18 Mathematics of Networks, April 2018

@iacopoiacopini

Contagion
on Networks

Simple Contagion

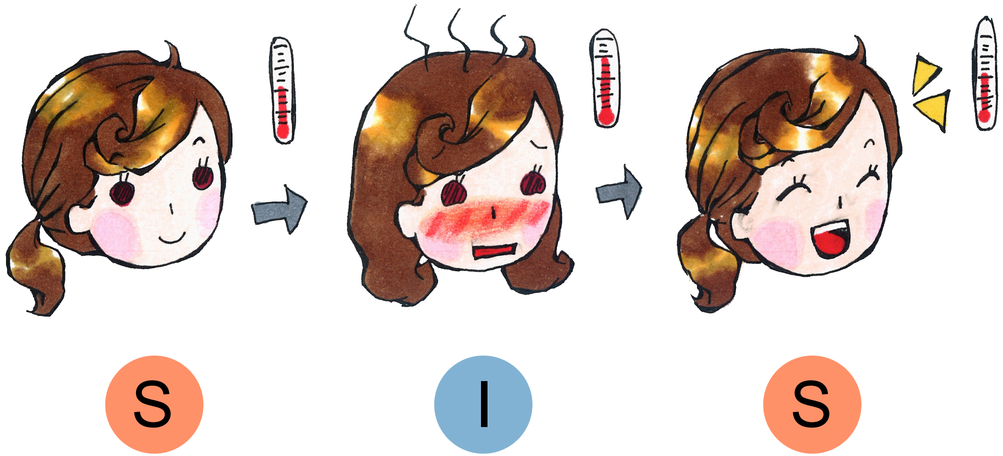
Spreading of infectious diseases

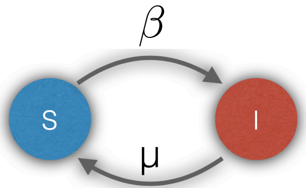


Artwork by Mi Jin Lee

Simple Contagion

Spreading of infectious diseases

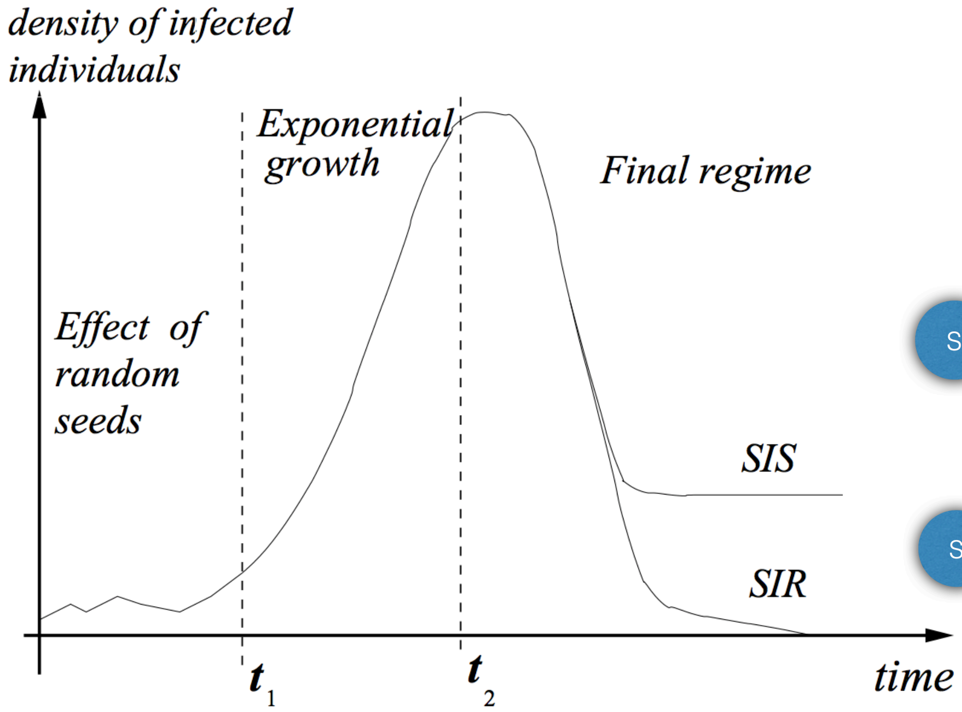


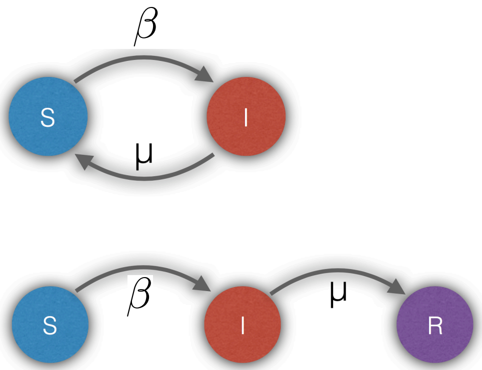


Artwork by Mi Jin Lee

Simple Contagion

Spreading of infectious diseases

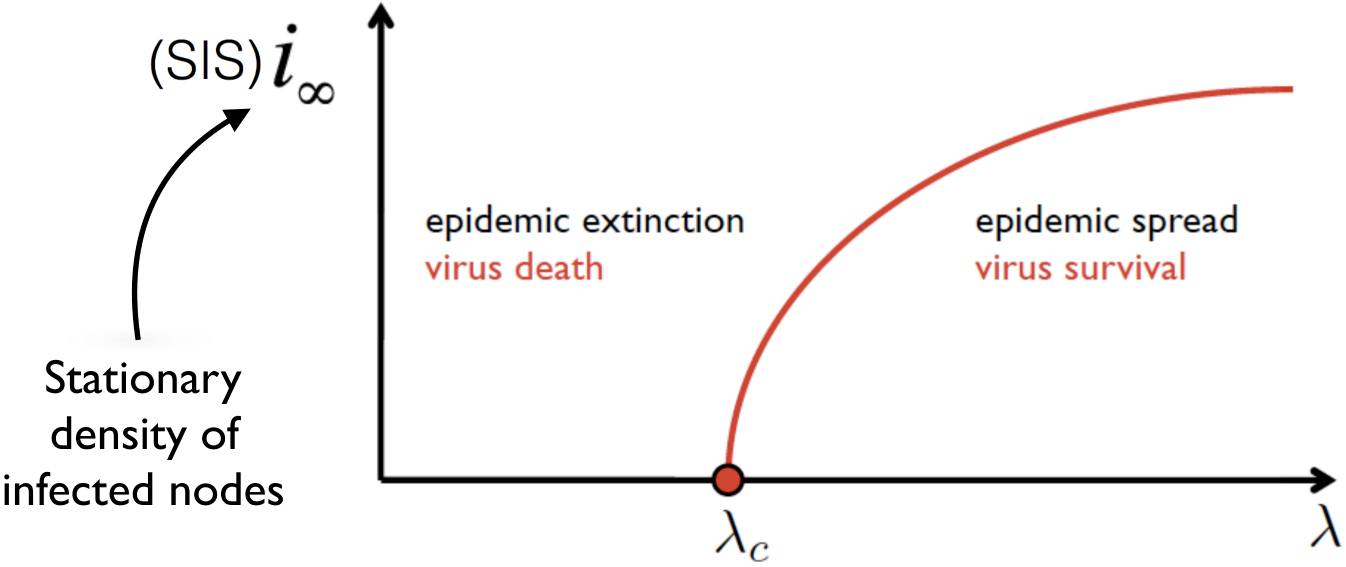




M. Barthélemy et al. J. Theor. Biol. (2005)

Simple Contagion

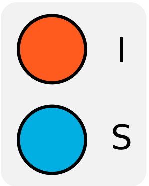
Spreading of infectious diseases

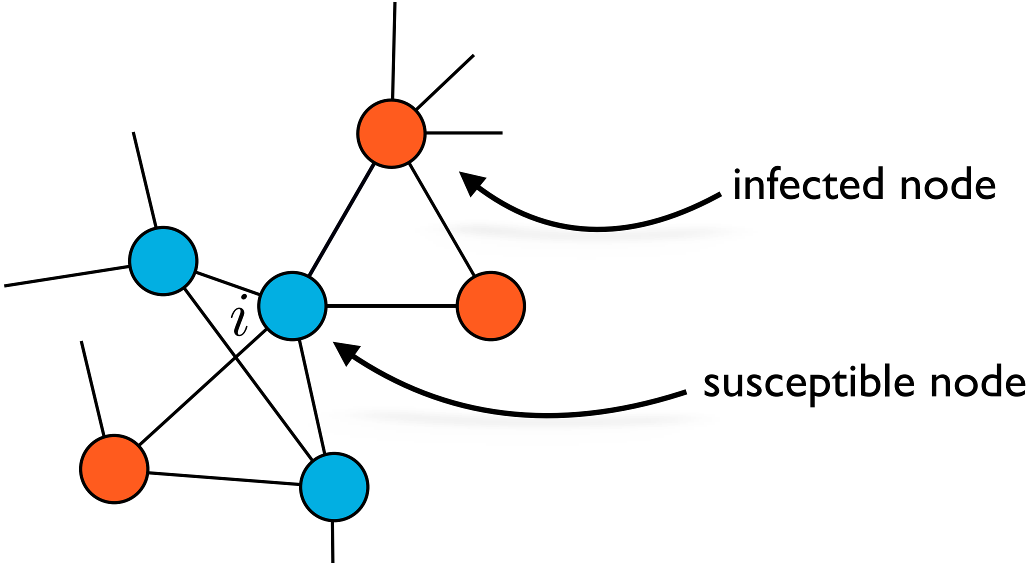


Epidemic threshold

Simple Contagion

Spreading of infectious diseases

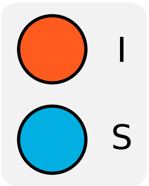




Transmission through **pairwise interactions** between infectious (I) and healthy (S) individuals

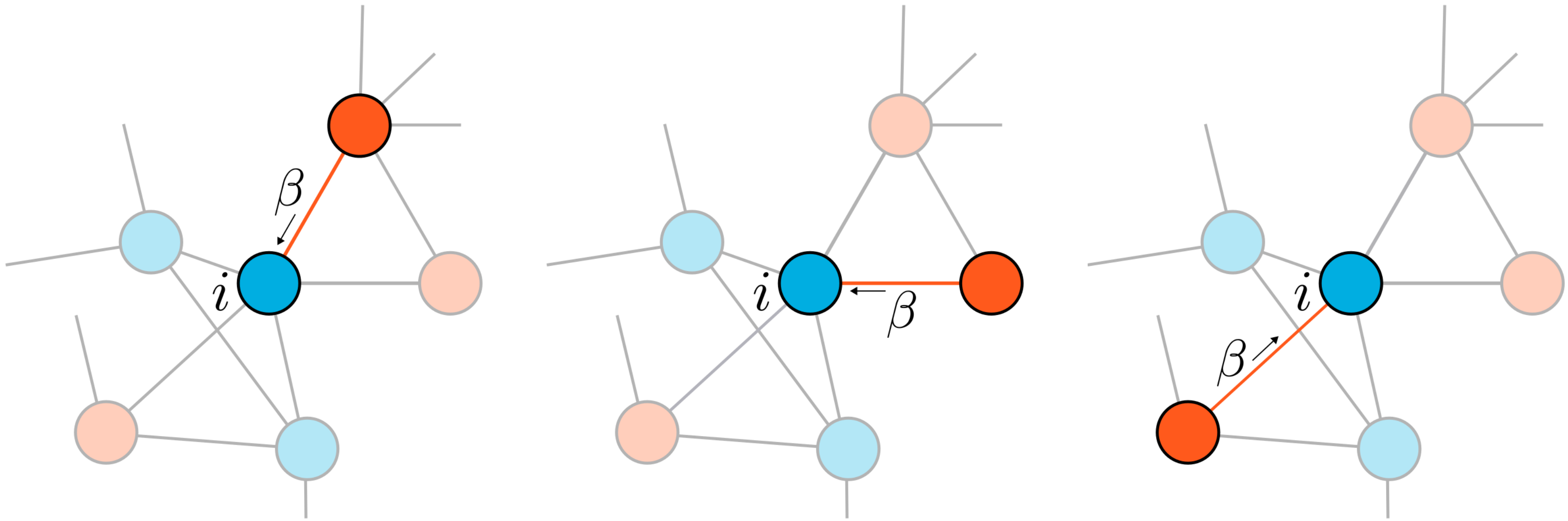
I+S $\longrightarrow$ 2I

β: probability of infection



Simple Contagion

Spreading of infectious diseases



Independent
sources of infection

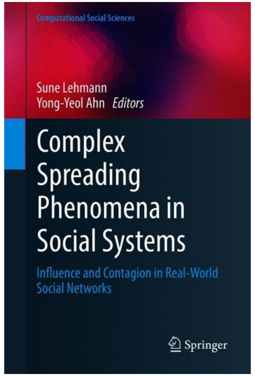
$I + S \longrightarrow 2I$

β: probability of infection

Social Contagion

Diffusion of information, behaviours, rumours, fads, beliefs, norms…

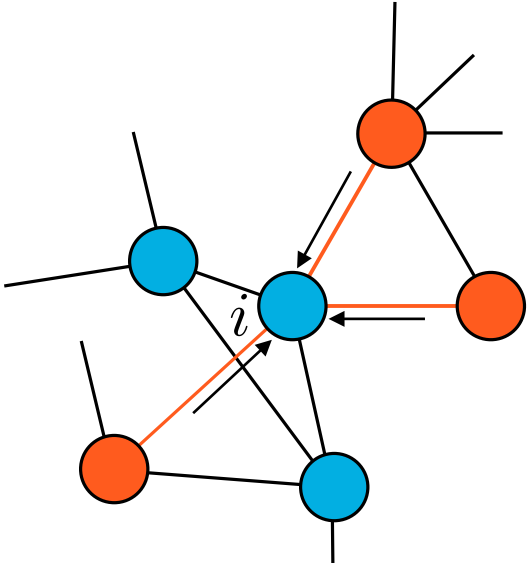
- Peer pressure
- Social influence
- Complex individual response to repeated exposures



D. Centola & M. Macy, Americ. J. Sociol. (2007); N. O. Todas & K. Lerman, Sci. Rep. (2014); D. Guilbeault et al., Springer (2018)

Complex Contagion

Social contagion



Multiple sources of activation are required for a transmission

D. Centola & M. Macy, Americ. J. Sociol. (2007); N. O. Todas & K. Lerman, Sci. Rep. (2014); D. Guilbeault et al., Springer (2018)

Complex Contagion

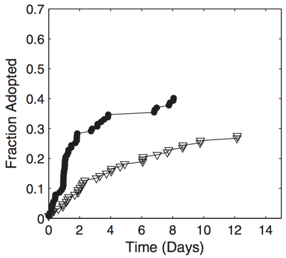
Evidence

REPORT

The Spread of Behavior in an Online Social Network Experiment

Damon Centola
+ See all authors and affiliations

Science 03 Sep 2010:
Vol. 329, Issue 5996, pp. 1194-1197
DOI: 10.1126/science.1185231

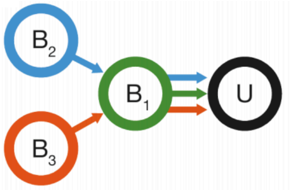


PLOS | ONE

RESEARCH ARTICLE

Evidence of complex contagion of information in social media: An experiment using Twitter bots

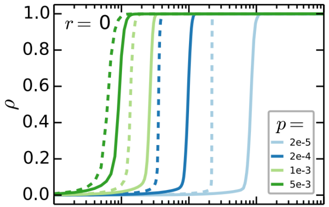
Bjarke Mønsted[1☯], Piotr Sapieżyński[1☯], Emilio Ferrara[2,3☯], Sune Lehmann[1☯*]



PRL 115, 218702 (2015) PHYSICAL REVIEW LETTERS week ending 20 NOVEMBER 2015

Kinetics of Social Contagion

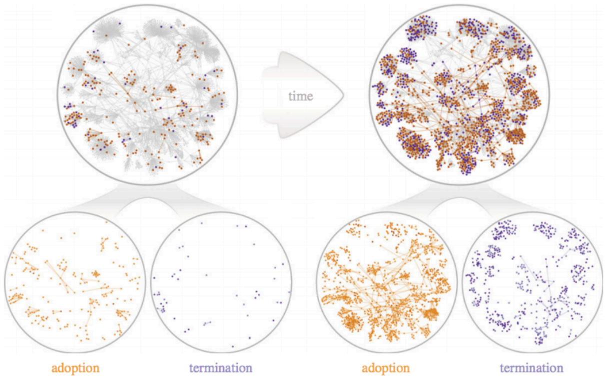
Zhongyuan Ruan,[1,2] Gerardo Iñiguez,[3,4] Márton Karsai,[5] and János Kertész[1,2,4,*]
[1]Center for Network Science, Central European University, H-1051 Budapest, Hungary
[2]Institute of Physics, Budapest University of Technology and Economics, H-1111 Budapest, Hungary
[3]Centro de Investigación y Docencia Económicas, Consejo Nacional de Ciencia y Tecnología, 01210 México D.F., Mexico
[4]Department of Computer Science, Aalto University School of Science, FI-00076 AALTO, Finland
[5]Laboratoire de l'Informatique du Parallélisme, INRIA-UMR 5668, IXXI, ENS de Lyon, 69364 Lyon, France



JOURNAL OF THE ROYAL SOCIETY Interface

Complex contagion process in spreading of online innovation

Márton Karsai[1,2,3,4], Gerardo Iñiguez[2], Kimmo Kaski[2,5] and János Kertész[2,6,7]

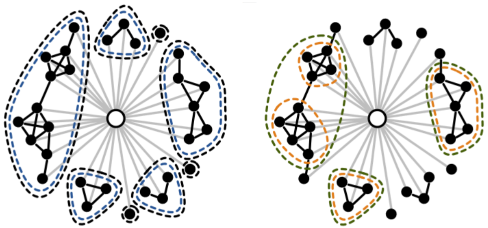


Structural diversity in social contagion

Johan Ugander, Lars Backstrom, Cameron Marlow, and Jon Kleinberg

PNAS April 17, 2012 109 (16) 5962-5966; https://doi.org/10.1073/pnas.1116502109

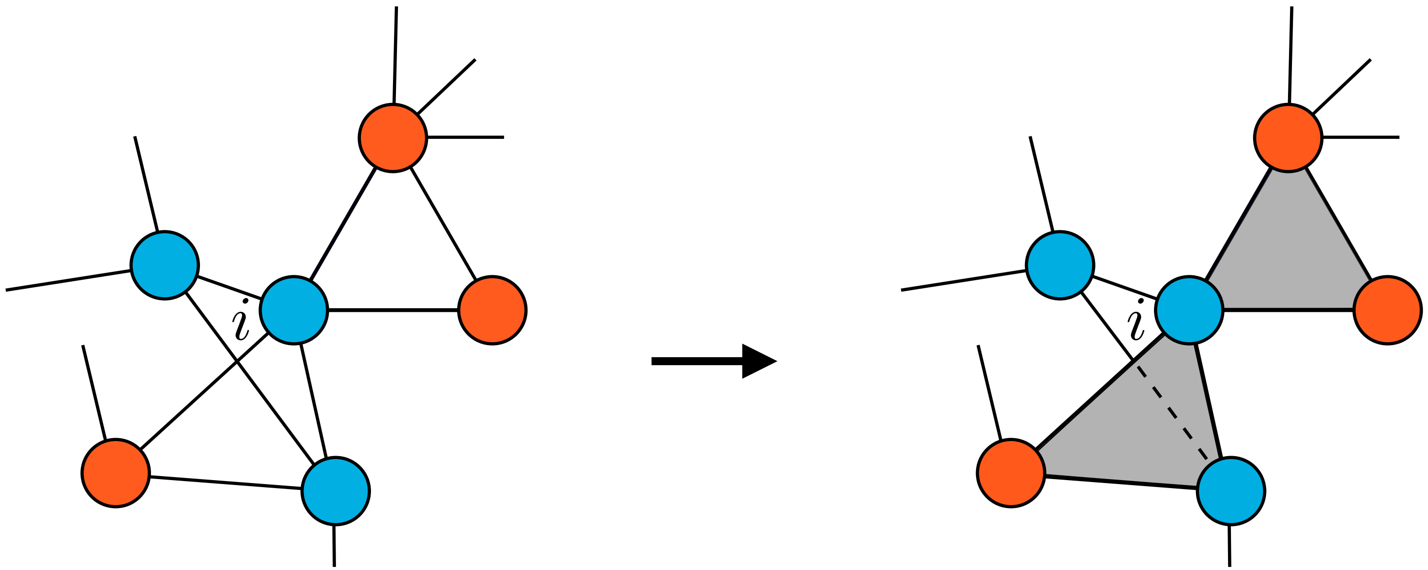
Edited by Ronald L. Graham, University of California at San Diego, La Jolla, CA, and approved February 21, 2012



Pairwise interactions


What about group interactions?

SIMPLicial Cont**AGION**

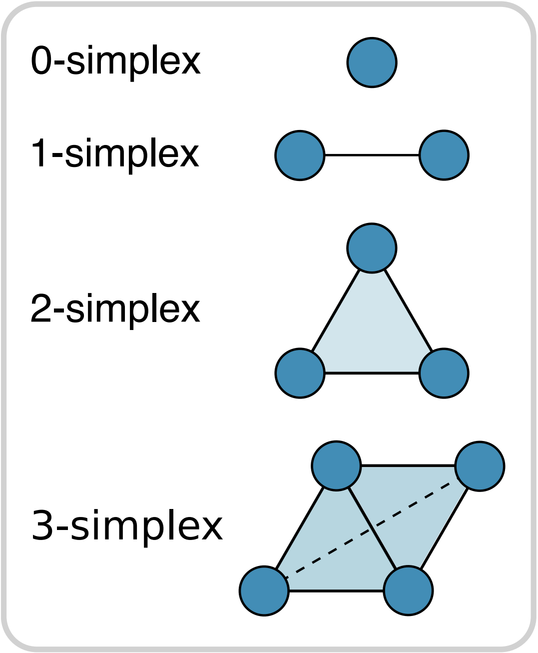


Network representation of the social structure

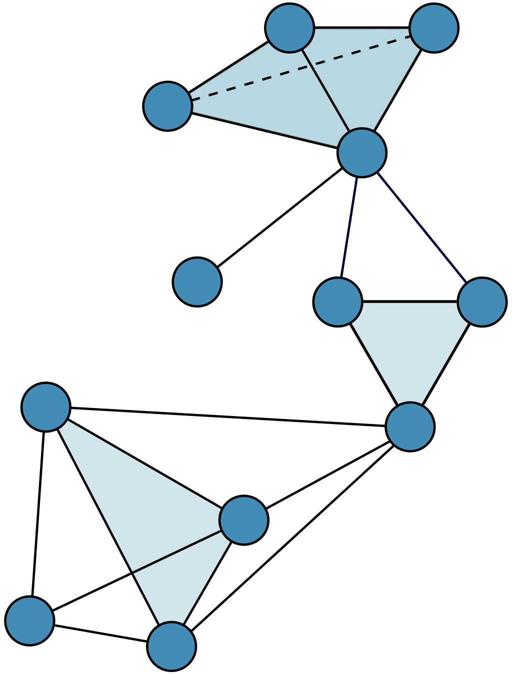
Simplicial complex

Simplicial complex

d-dimensional
group interactions

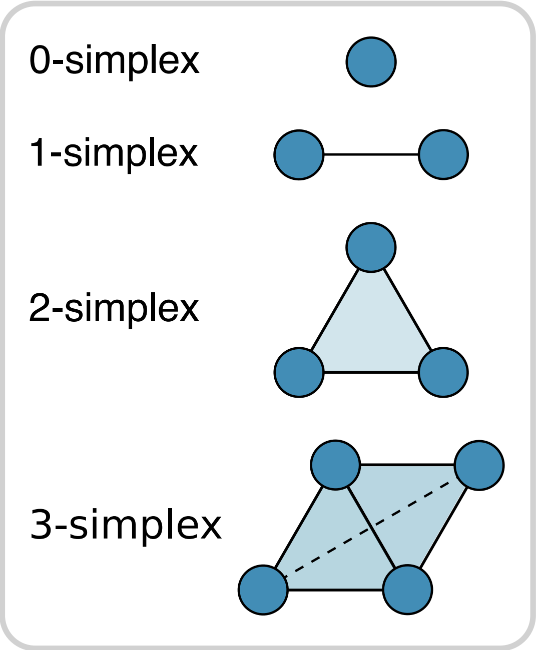


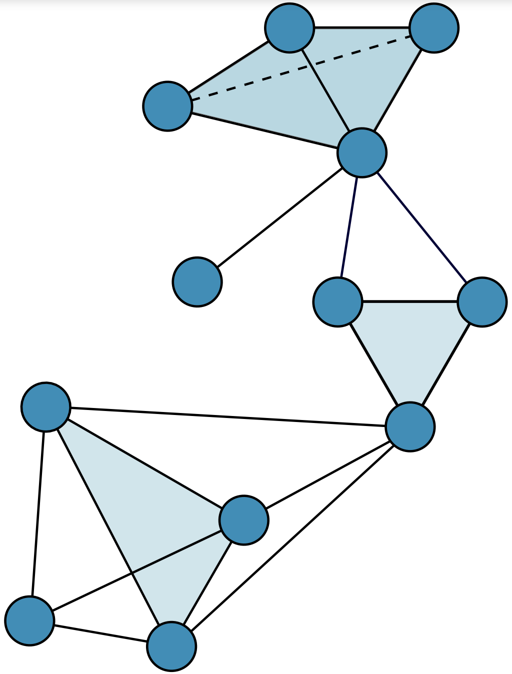
Social structure:
simplicial complex



Simplicial complex

A simplicial complex $\mathscr{K}$ on a given set of vertices $\mathscr{V}$, with $|\mathscr{V}| = N$, is a collection of simples with the extra requirement that if a simplex $\sigma \in \mathscr{K}$, then all the sub-simplices $\nu \subset \sigma$ built from the subset of σ are also contained in $\mathscr{K}$.





Simplicial complex

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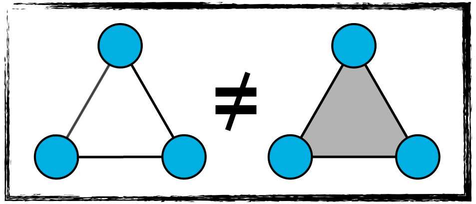
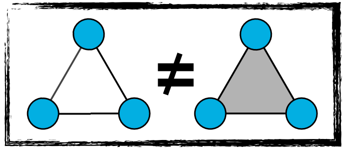
Notice that



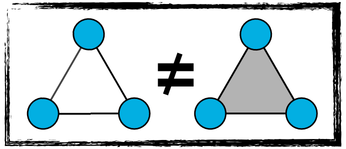
Simplicial complex

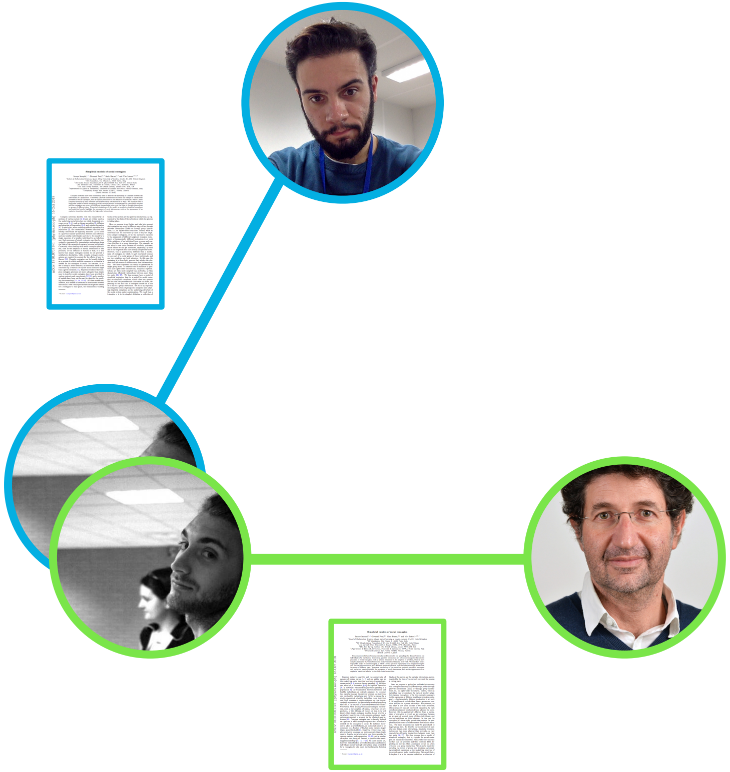
Interaction: co-authorship



Simplicial complex

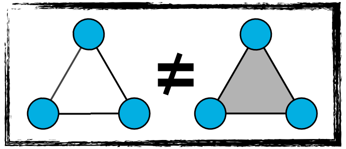
Interaction: co-authorship

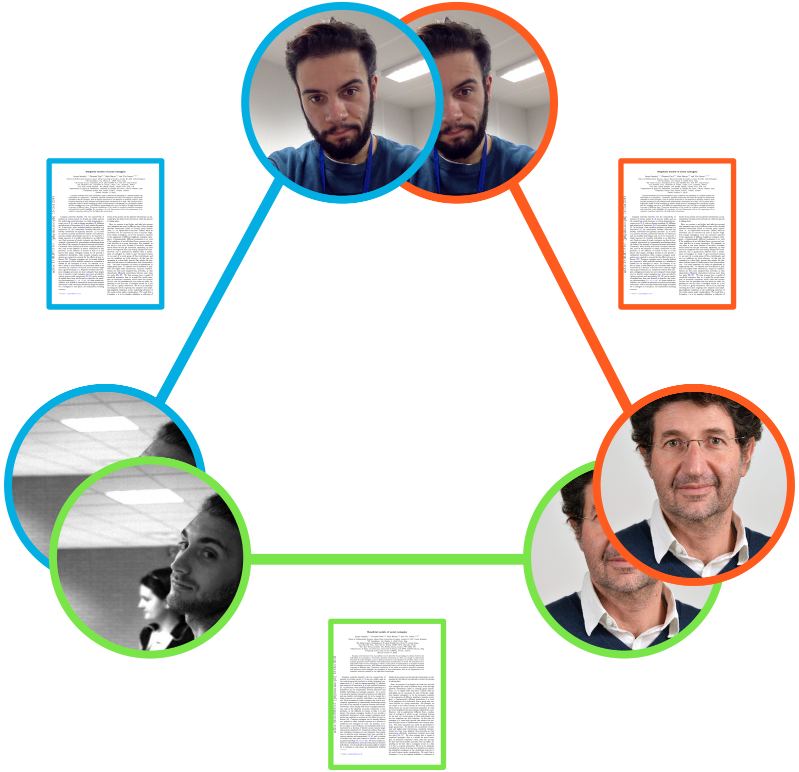




Simplicial complex

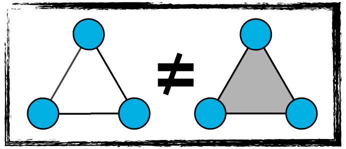
Interaction: co-authorship

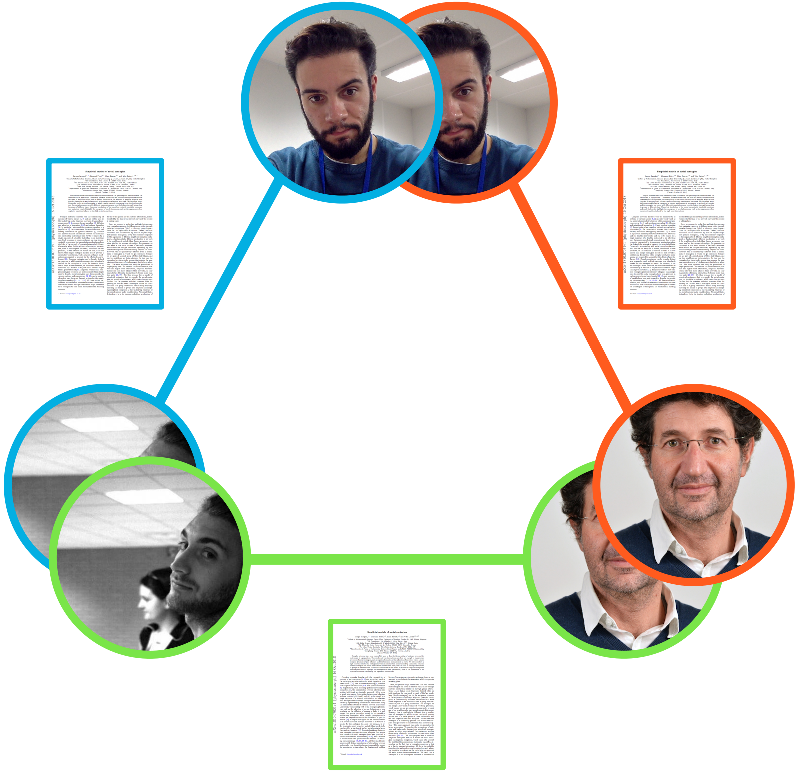


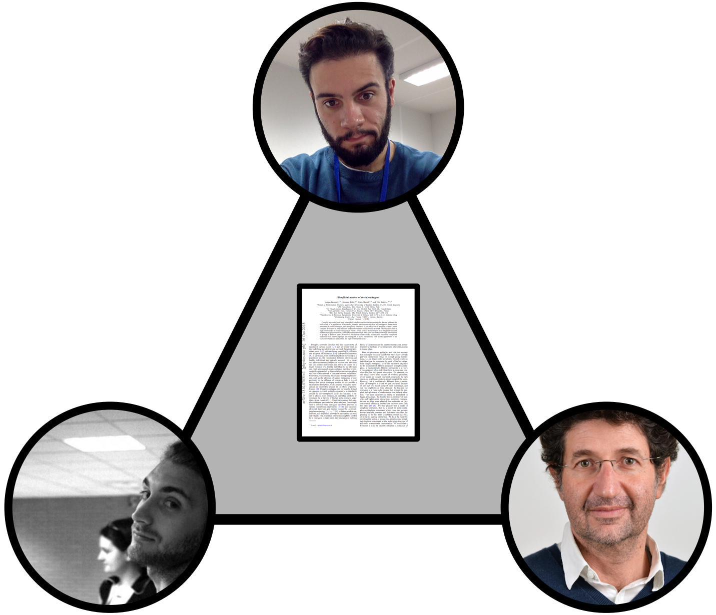


Simplicial complex

Interaction: co-authorship



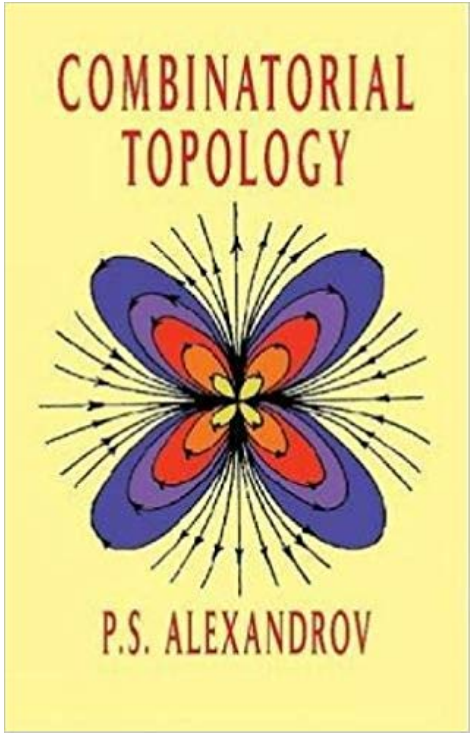




Sorry Alain!

Simplicial complexes

Not a new idea



Renewed interest among the complex systems community

European Journal of Physics

PAPER

Simplicial complexes and complex systems

Vsevolod Salnikov[1], Daniele Cassese[1,2] and Renaud Lambiotte[3]

Published 14 November 2018 • © 2018 European Physical Society
European Journal of Physics, Volume 40, Number 1
Focus on Complexity

NETWORK NEURO SCIENCE

FOCUS

The importance of the whole: Topological data analysis for the network neuroscientist

Ann E. Sizemore[1], Jennifer E. Phillips-Cremins[1], Robert Ghrist[2], and Danielle S. Bassett[1,3,4,5]

[1]Department of Bioengineering, School of Engineering and Applied Sciences, University of Pennsylvania, Philadelphia, USA
[2]Department of Mathematics, College of Arts and Sciences, University of Pennsylvania, Philadelphia, USA
[3]Department of Physics & Astronomy, College of Arts and Sciences, University of Pennsylvania, Philadelphia, USA
[4]Department of Electrical & Systems Engineering, School of Engineering and Applied Sciences, University of Pennsylvania, Philadelphia, USA
[5]Department of Neurology, Perelman School of Medicine, University of Pennsylvania, Philadelphia, USA

Simplicial complexes

Describing the architecture of complex networks

OPEN ACCESS Freely available online PLOS | ONE

Topological Strata of Weighted Complex Networks

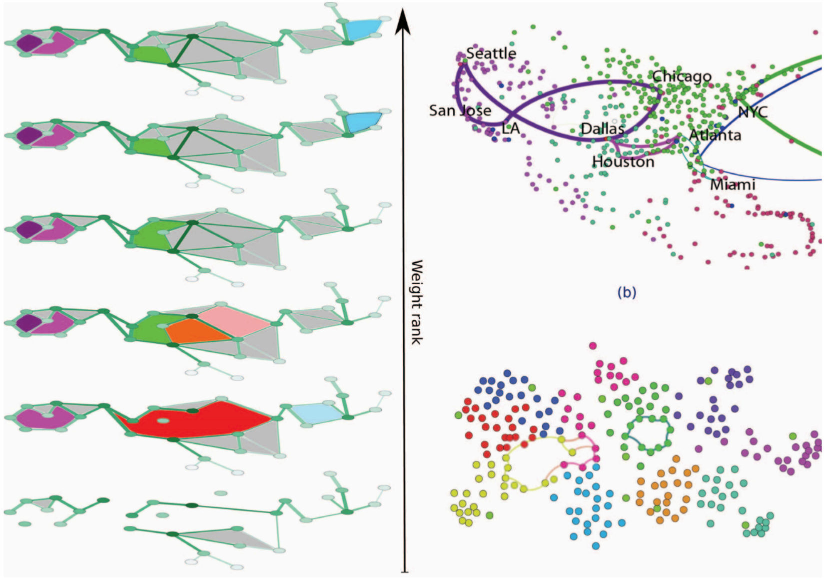
Giovanni Petri[1]*, Martina Scolamiero[1,2], Irene Donato[1,3], Francesco Vaccarino[1,3]

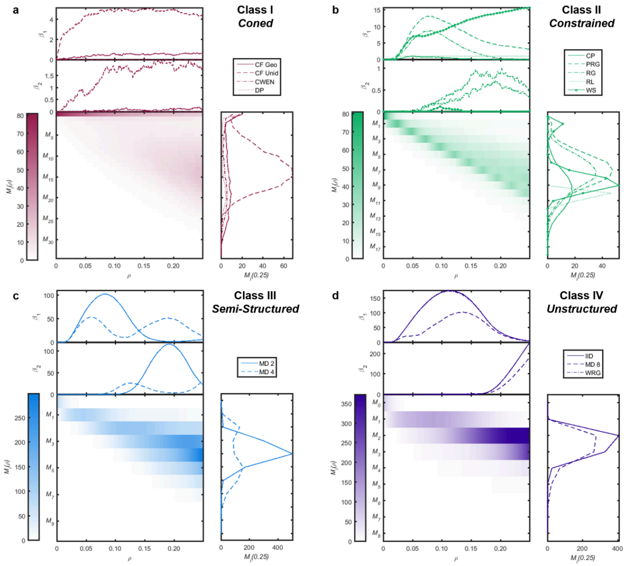
1 ISI Foundation, Torino, Italy, 2 Dipartimento di Ingegneria Gestionale e della Produzione, Politecnico di Torino, Torino, Italy, 3 Dipartimento di Scienze Matematiche, Politecnico di Torino, Torino, Italy

Classification of weighted networks through mesoscale homological features

Ann Sizemore, Chad Giusti, Danielle S. Bassett

Journal of Complex Networks, Volume 5, Issue 2, June 2017, Pages 245–273,





Simplicial complexes

Describing the architecture of functional and structural brain networks

JOURNAL OF THE ROYAL SOCIETY Interface
rsif.royalsocietypublishing.org

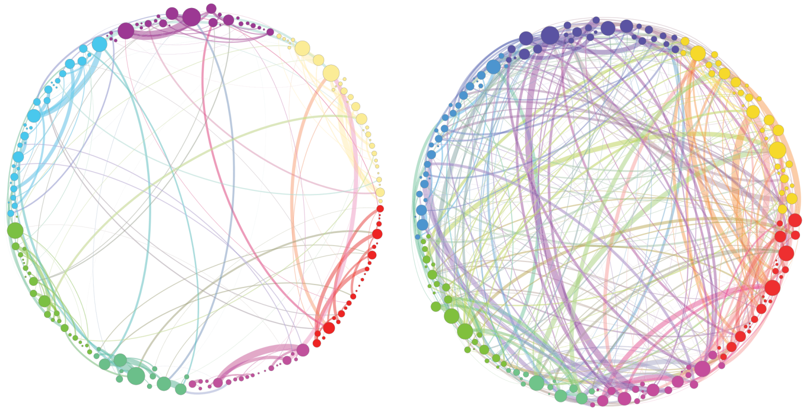
Homological scaffolds of brain functional networks

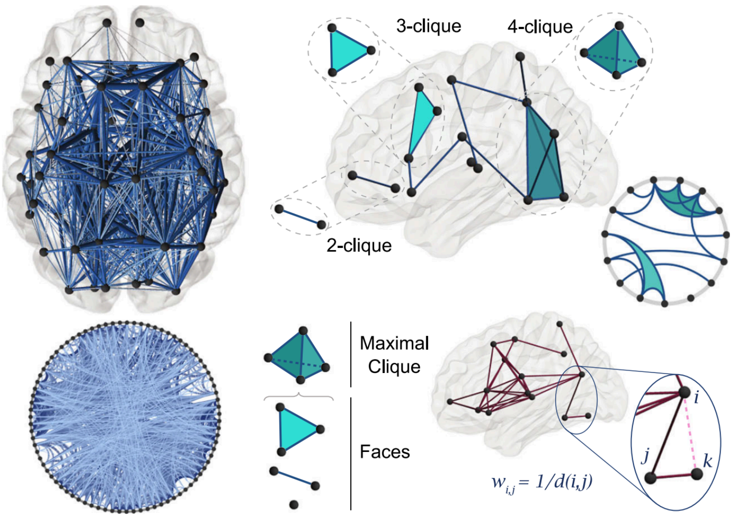
G. Petri[1], P. Expert[2], F. Turkheimer[2], R. Carhart-Harris[3], D. Nutt[3], P. J. Hellyer[4] and F. Vaccarino[1,5]

J Comput Neurosci (2018) 44:115–145
https://doi.org/10.1007/s10827-017-0672-6

Cliques and cavities in the human connectome

Ann E. Sizemore[1,2] · Chad Giusti[1] · Ari Kahn[1,3] · Jean M. Vettel[1,3,4] · Richard F. Betzel[1] · Danielle S. Bassett[1,5]





Simplicial complexes

Describing the architecture of semantic and co-authorship networks

ARTICLES
https://doi.org/10.1038/s41562-018-0422-4
nature human behaviour

Knowledge gaps in the early growth of semantic feature networks

Ann E. Sizemore[1], Elisabeth A. Karuza[2], Chad Giusti[3] and Danielle S. Bassett[1,4,5,6*]

Patania et al. EPJ Data Science (2017) 6:18
DOI 10.1140/epjds/s13688-017-0114-8
EPJ.org
EPJ Data Science
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REGULAR ARTICLE
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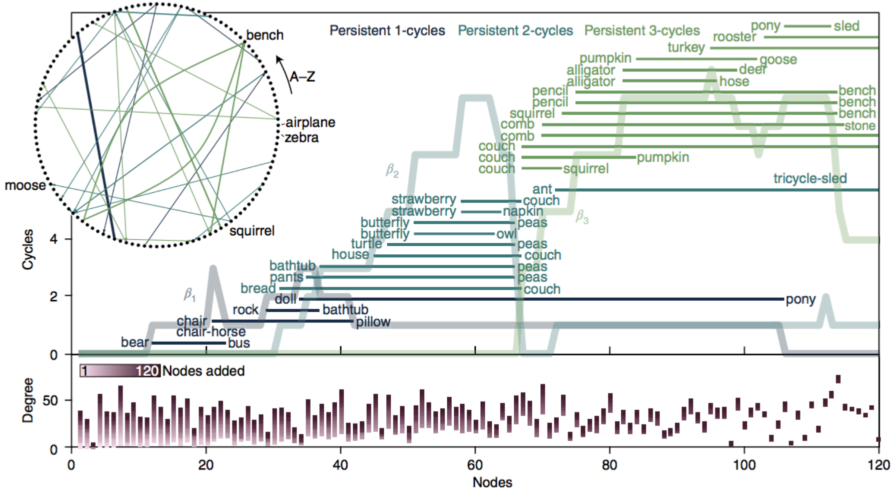
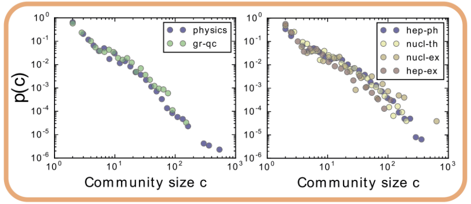
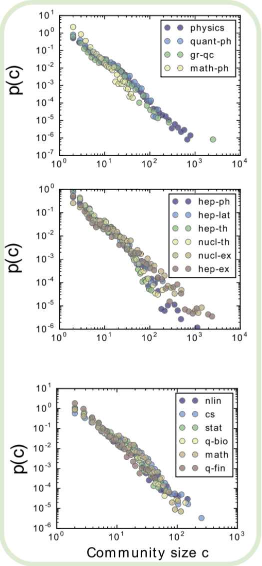
The shape of collaborations

Alice Patania[1,2*], Giovanni Petri[1] and Francesco Vaccarino[1,2]



	gr-qc	hep-ex	hep-ph	nucl-ex	nucl-th	physics	q-bio
gr-qc	0.01	0.00	0.00	0.01	0.09	0.00	0.07
hep-ex	0.04	0.00	0.00	0.00	0.07	0.00	0.05
hep-ph	0.02	0.01	0.02	0.01	0.08	0.00	0.06
nucl-ex	0.00	0.04	0.02	0.06	0.05	0.01	0.03
nucl-th	0.02	0.11	0.08	0.03	0.04	0.08	0.00
physics	0.03	0.00	0.00	0.03	0.10	0.00	0.06
q-bio	0.09	0.21	0.18	0.10	0.02	0.20	0.00

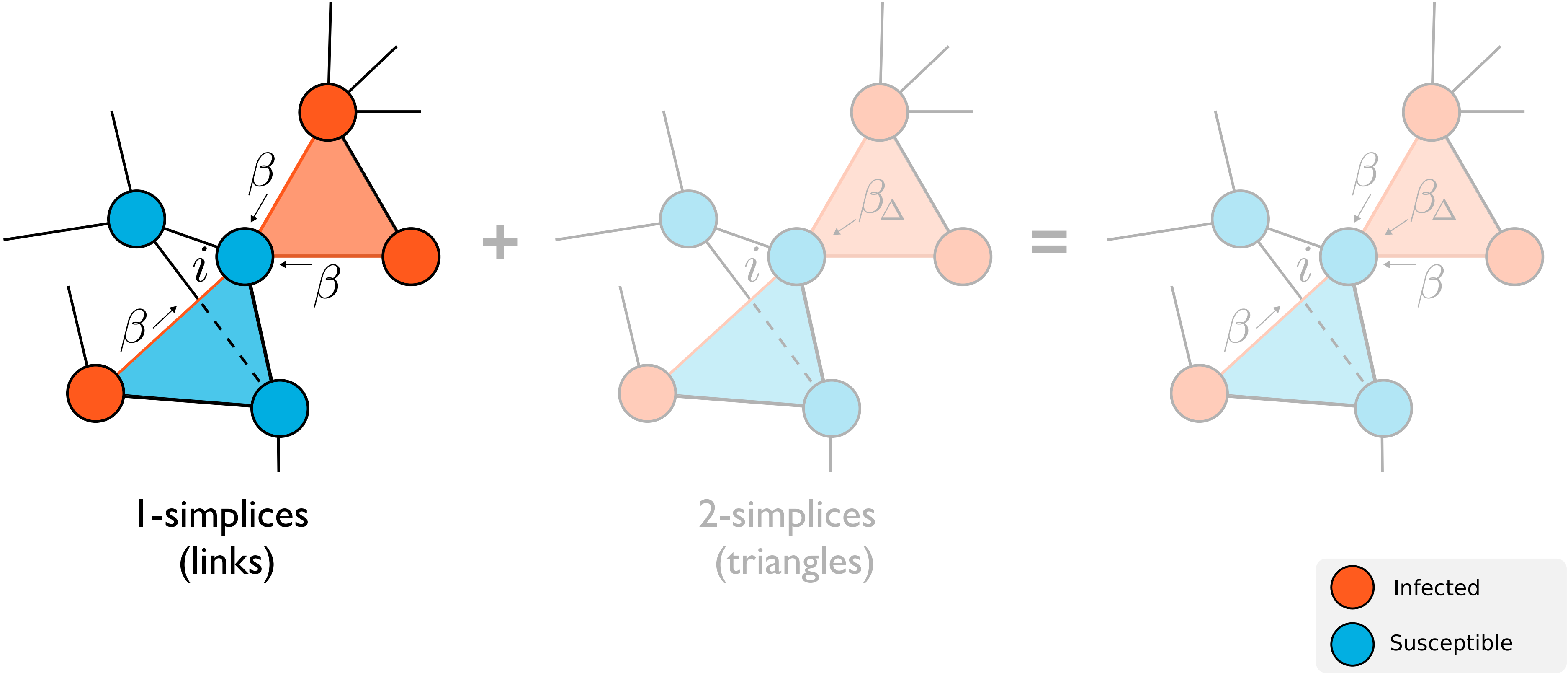




The Simplicial Contagion Model

SIMPLicial ContAGION

The Model (D=2)



SIMPLicial contAGION

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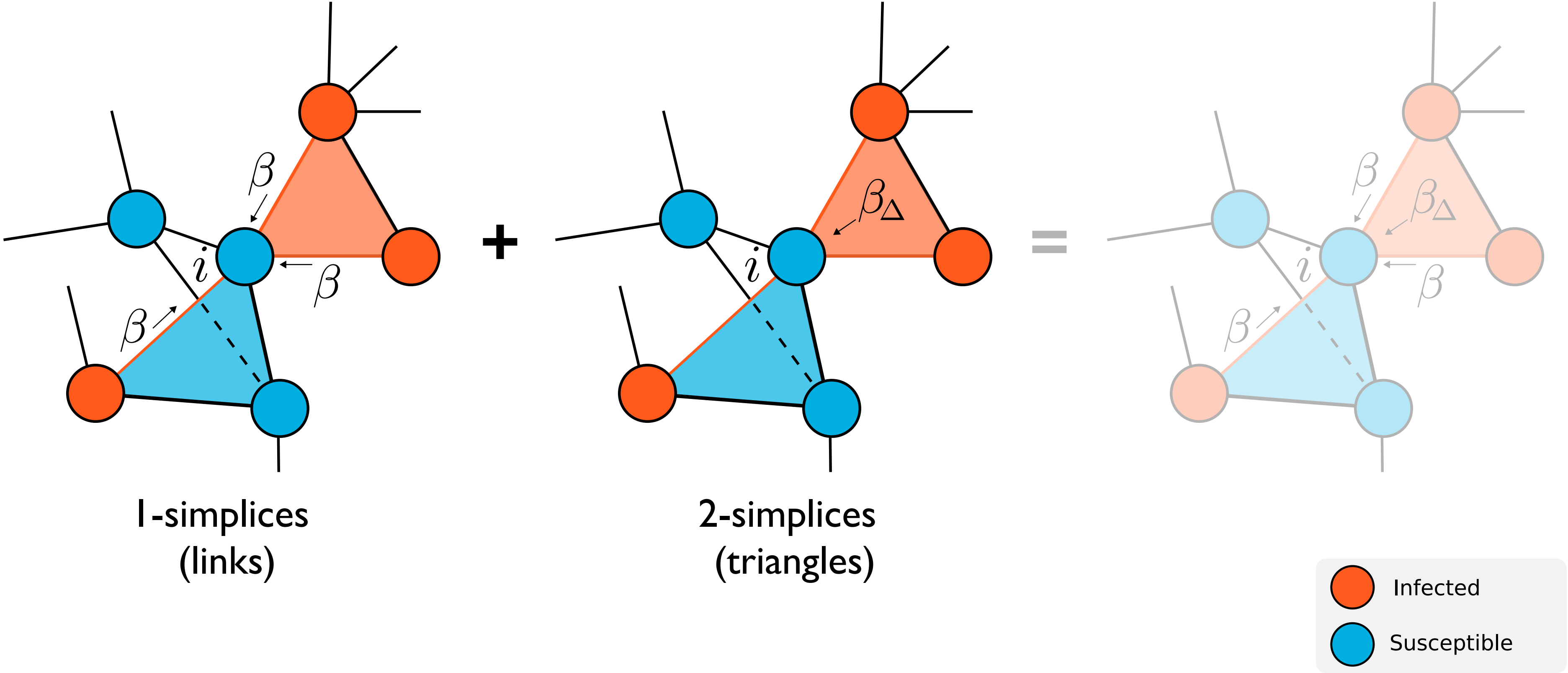

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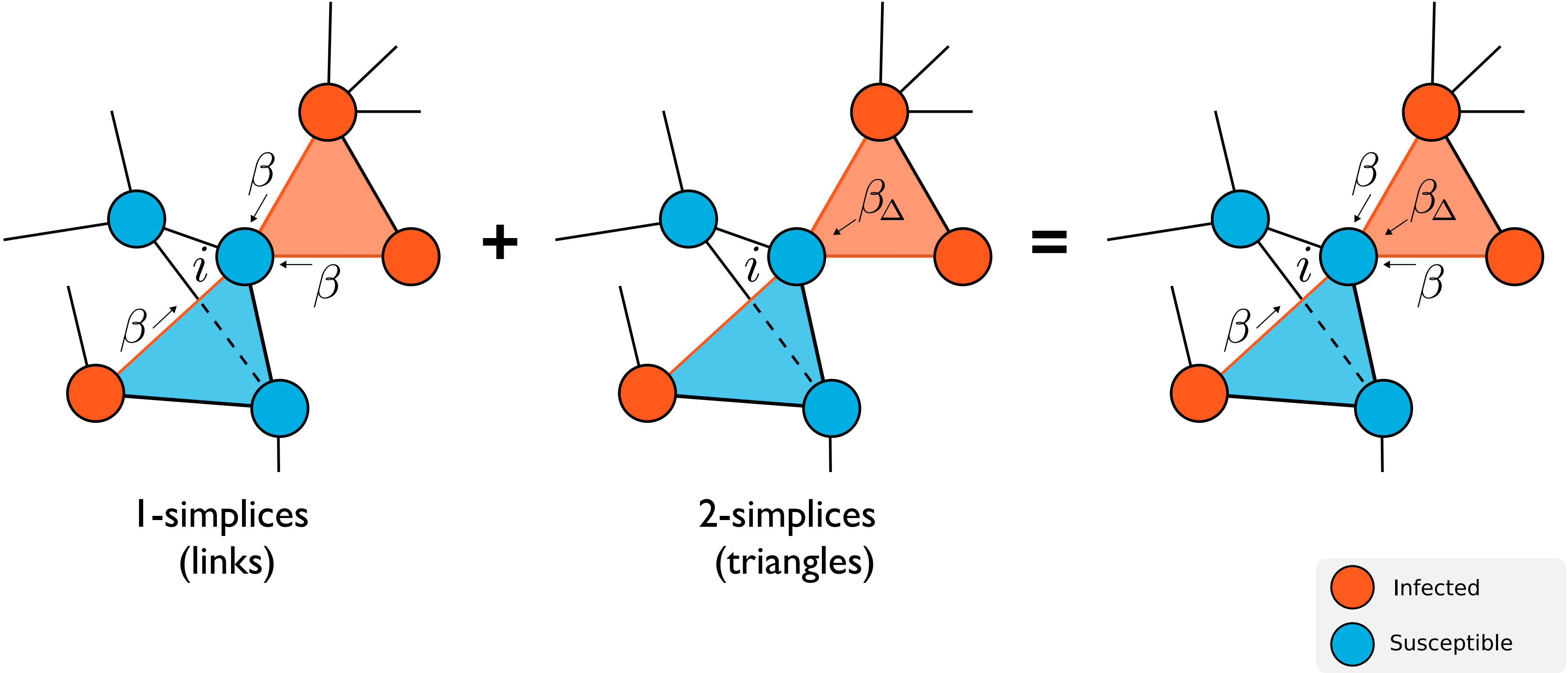

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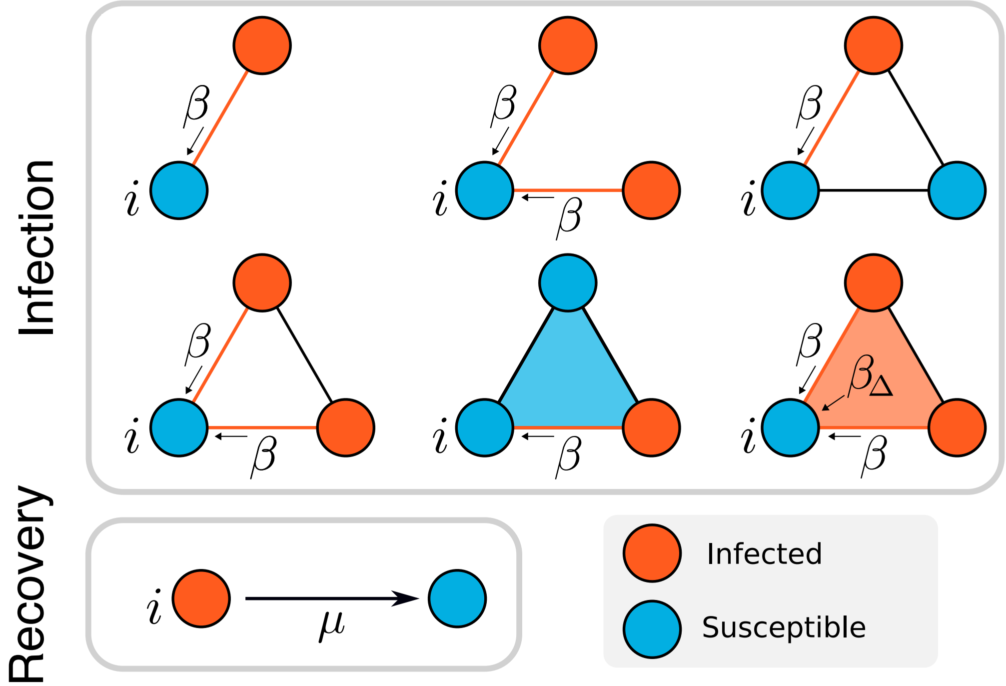
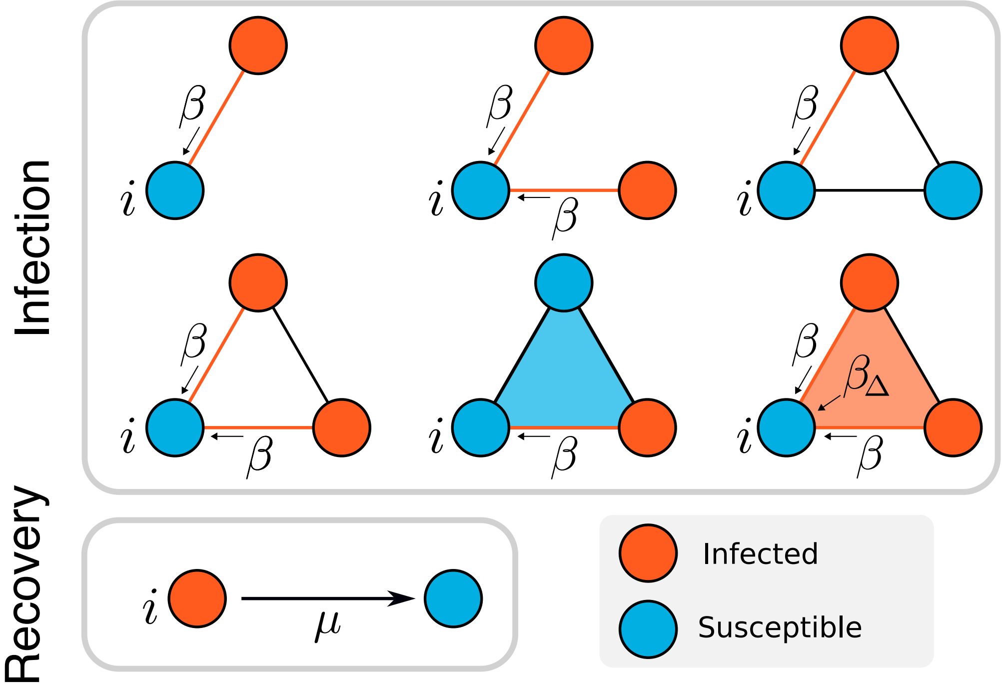


SIMPLicial contAGION

The Model (D=2)

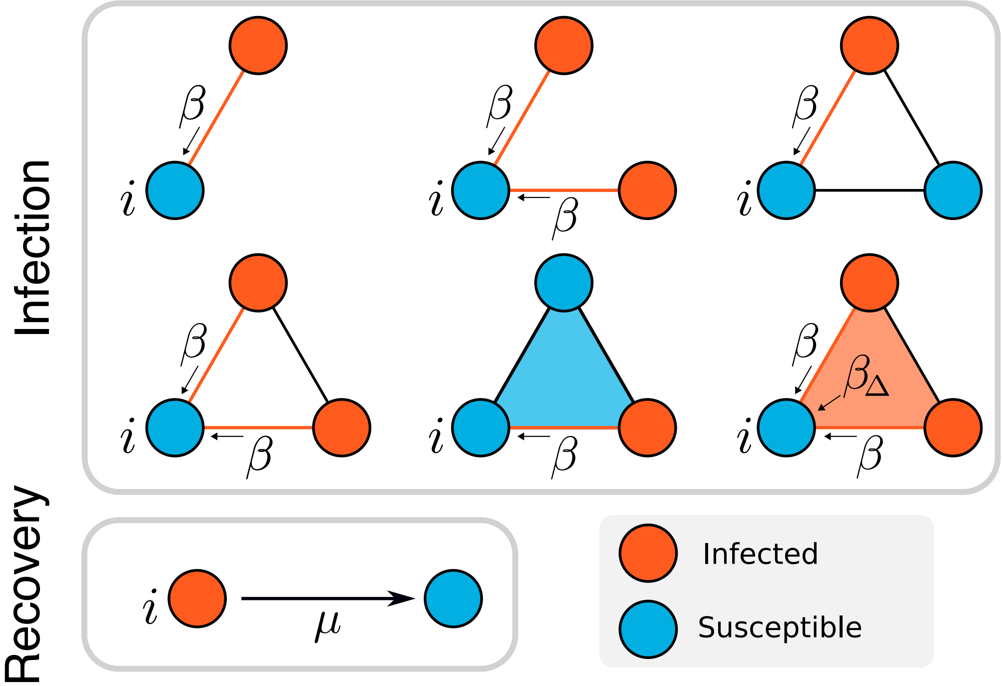


dynamical state variable

$$x_i(t) \in \{0,1\}$$

SIMPLicial contAGION

The Model (D=2)



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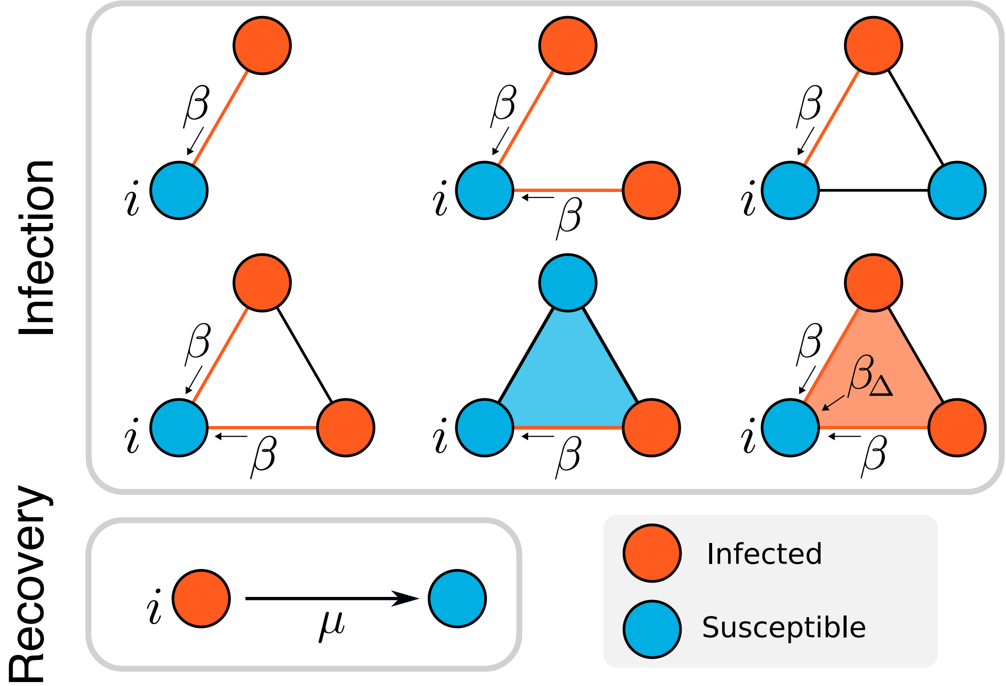
SIMPLicial contAGION

The Model (D=2)



dynamical state variable

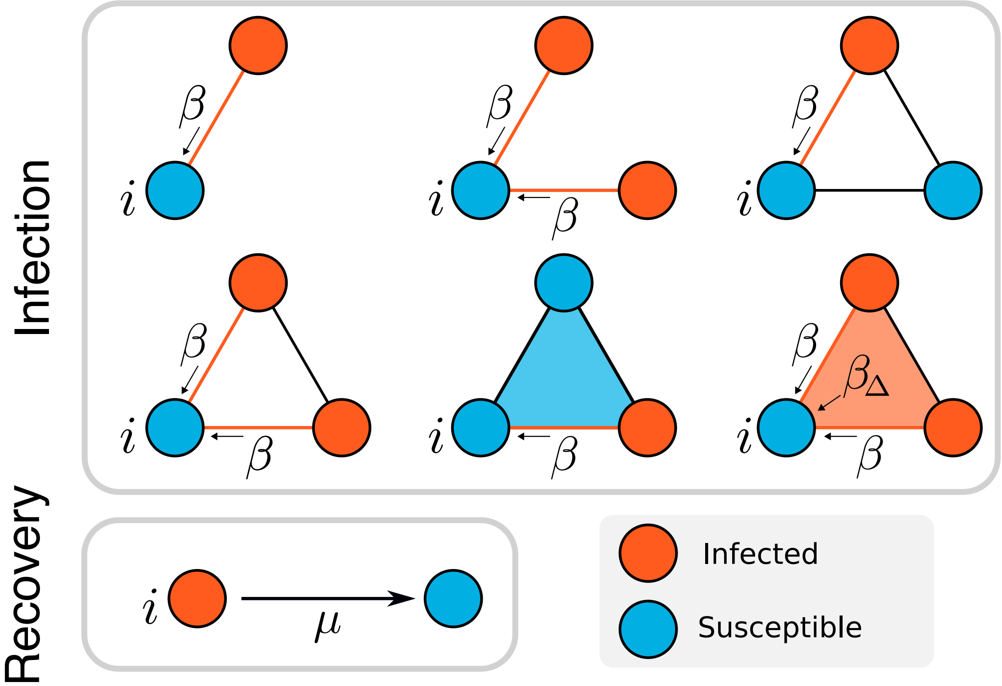
$$x_i(t) \in \{0,1\}$$

control parameters

$$\boxed{\begin{aligned}\lambda &= \beta\langle k\rangle/\mu \\ \lambda_\Delta &= \beta_\Delta\langle k_\Delta\rangle/\mu\end{aligned}}$$

SIMPLicial ContAGION

The Model (D=2)



dynamical state variable

$$x_i(t) \in \{0,1\}$$

control parameters

$$\lambda = \beta\langle k\rangle/\mu$$

$$\lambda_\Delta = \beta_\Delta\langle k_\Delta\rangle/\mu$$

k_w : generalised (simplicial) degree

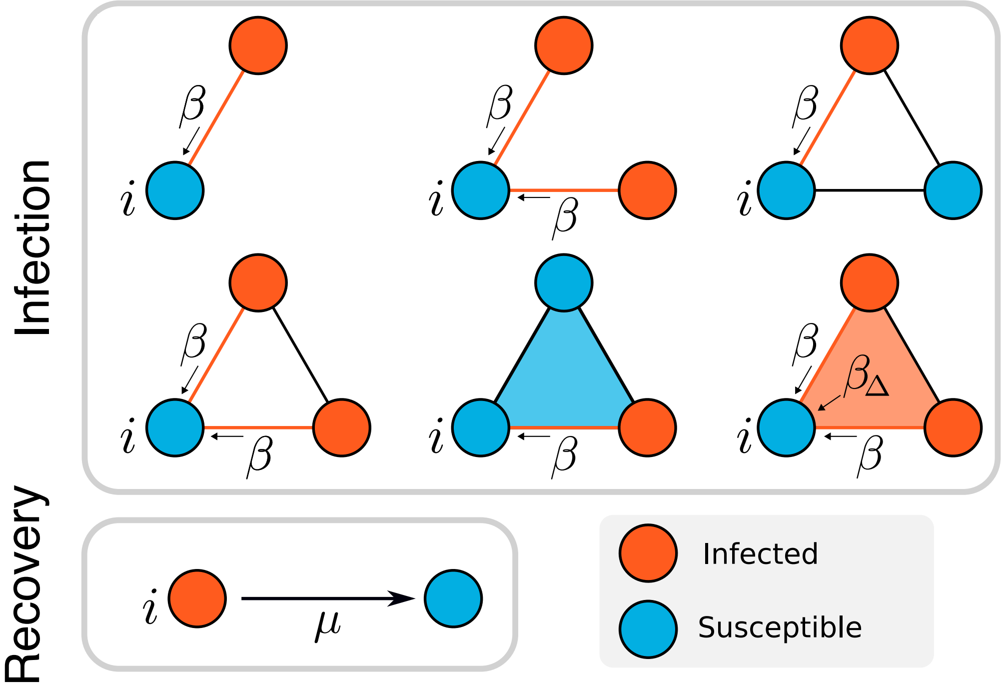
$$\langle k_1\rangle = \langle k\rangle$$

$$\langle k_2\rangle = \langle k_\Delta\rangle$$

O.T. Courtney & G. Bianconi, PRE (2016)

SIMPLicial ContAGION

The Model (D=2)



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control parameters

$$\boxed{\begin{aligned}\lambda &= \beta\langle k\rangle/\mu \\ \lambda_\Delta &= \beta_\Delta\langle k_\Delta\rangle/\mu\end{aligned}}$$

macroscopic order parameter

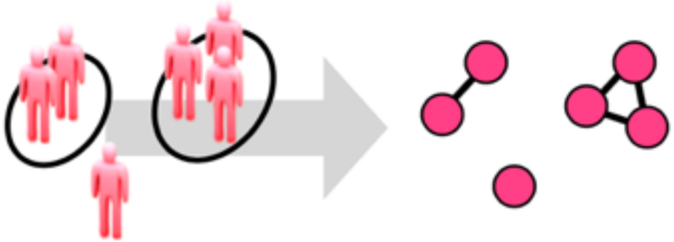
$$\rho(t) = \frac{1}{N}\sum_i x_i(t)$$

Empirical Social Structures

Gathering data

The SocioPatterns collaboration

what are the statistical and dynamical properties of the networks of contact and co-presence of people in social interaction?

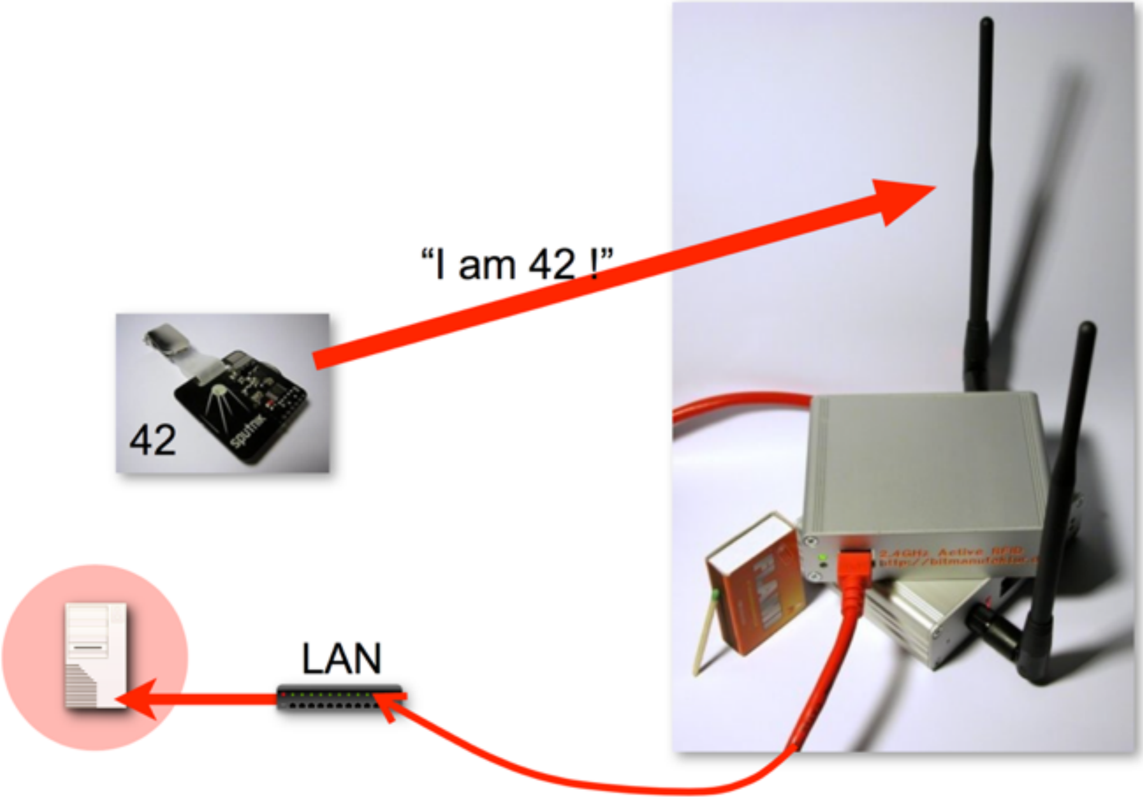


fine-grained **spatial** (~ m) and **temporal** (<min) resolution

Thanks to D. Paolotti

Gathering data

The SocioPatterns collaboration

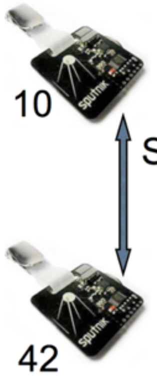


Thanks to D. Paolotti

SocioPatterns

Gathering data

The SocioPatterns collaboration



Short distance (~1-2m)
Exchange of very low power data packets

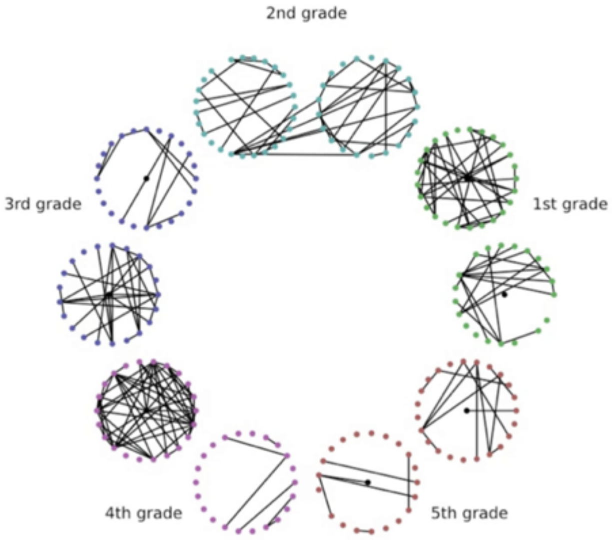
- Two power levels => 2 detection ranges
- Face to face situation
- Statistical detection => 20s time resolution
- Small,
- Scalable

Thanks to D. Paolotti

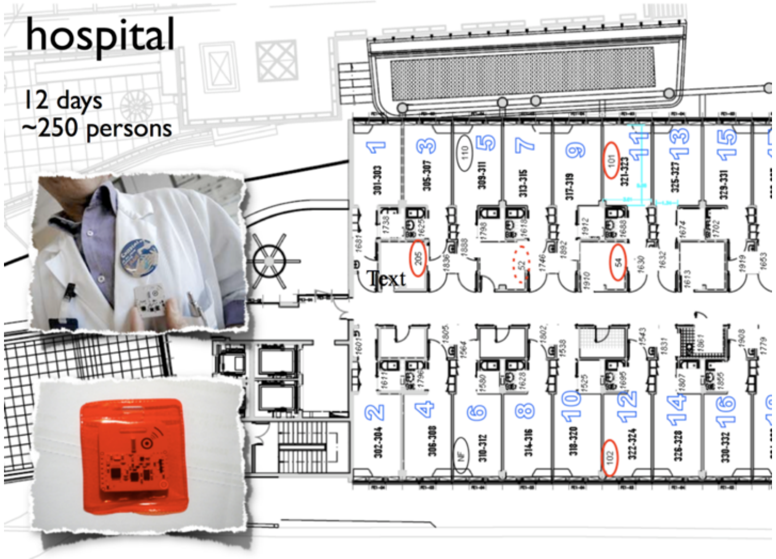
SocioPatterns

Gathering data

The SocioPatterns collaboration



Primary school

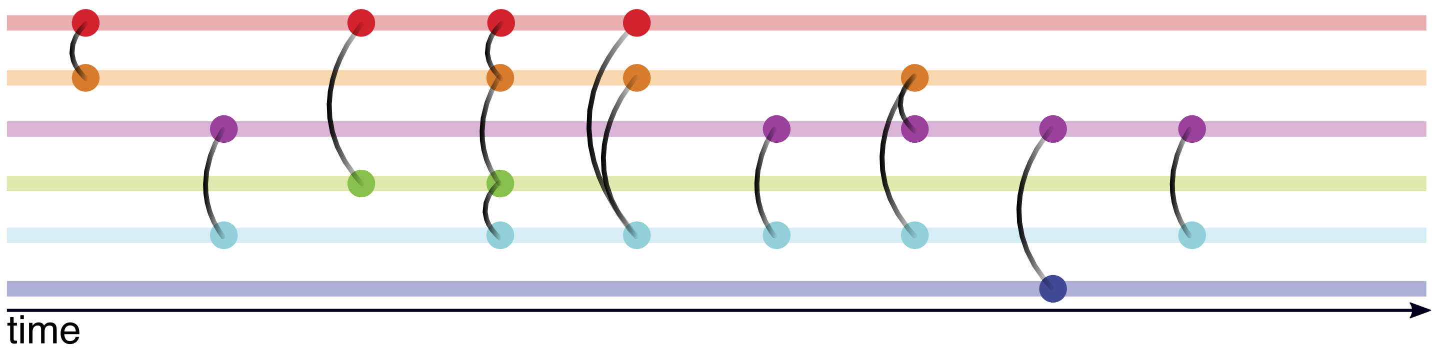


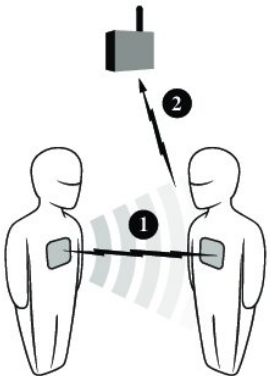
Hospital

J. Stehle et al. PLoS One (2011)

Real world simplicial complexes

High-resolution proximity data





M. Génois and A. Barrat, EPJ Data Science (2018)

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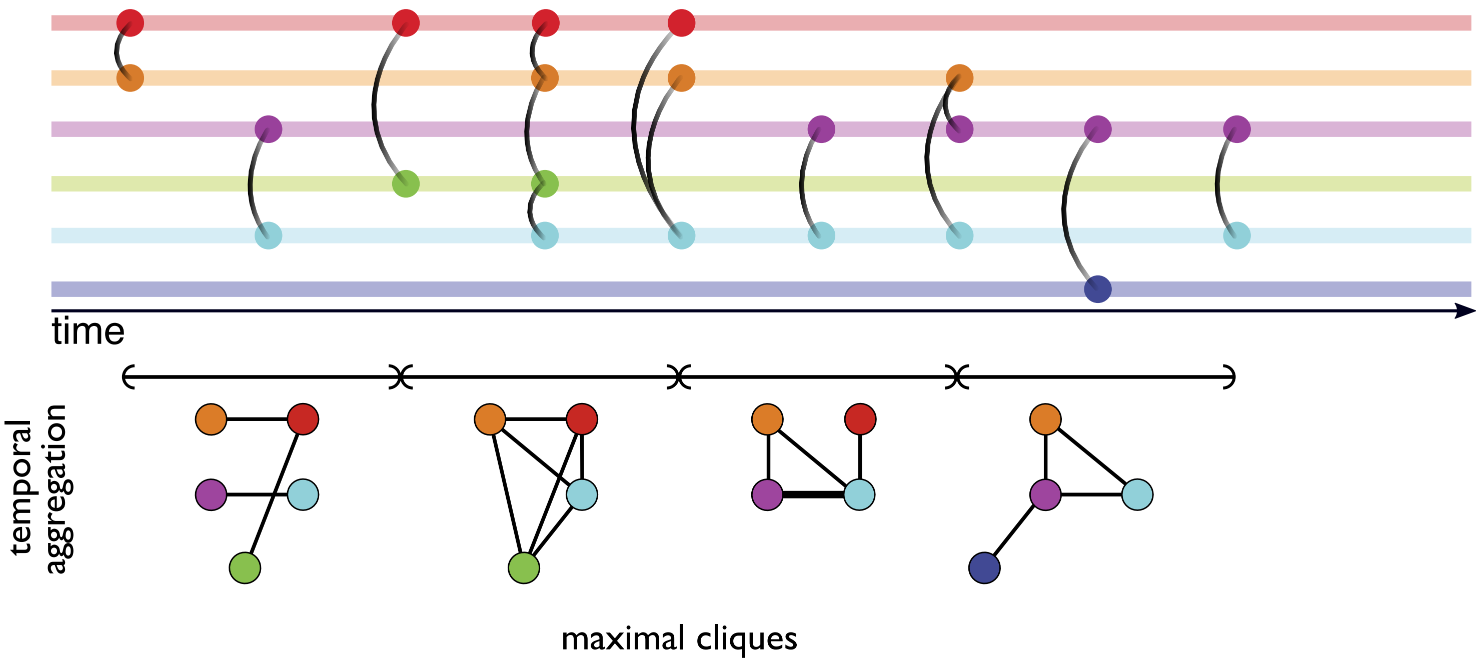

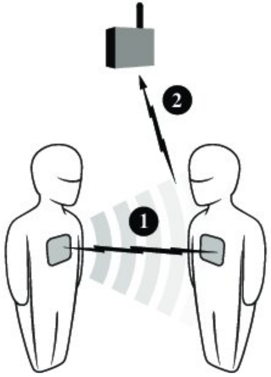

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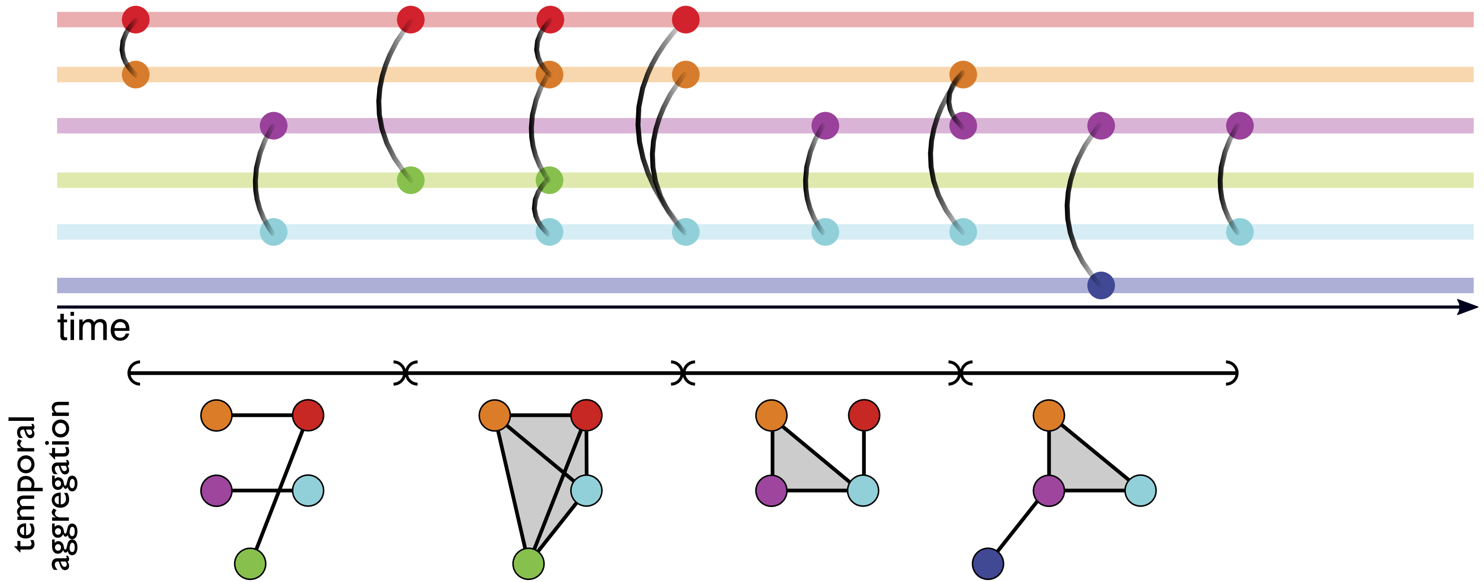
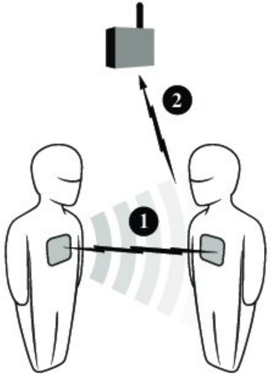


simplicial complexes



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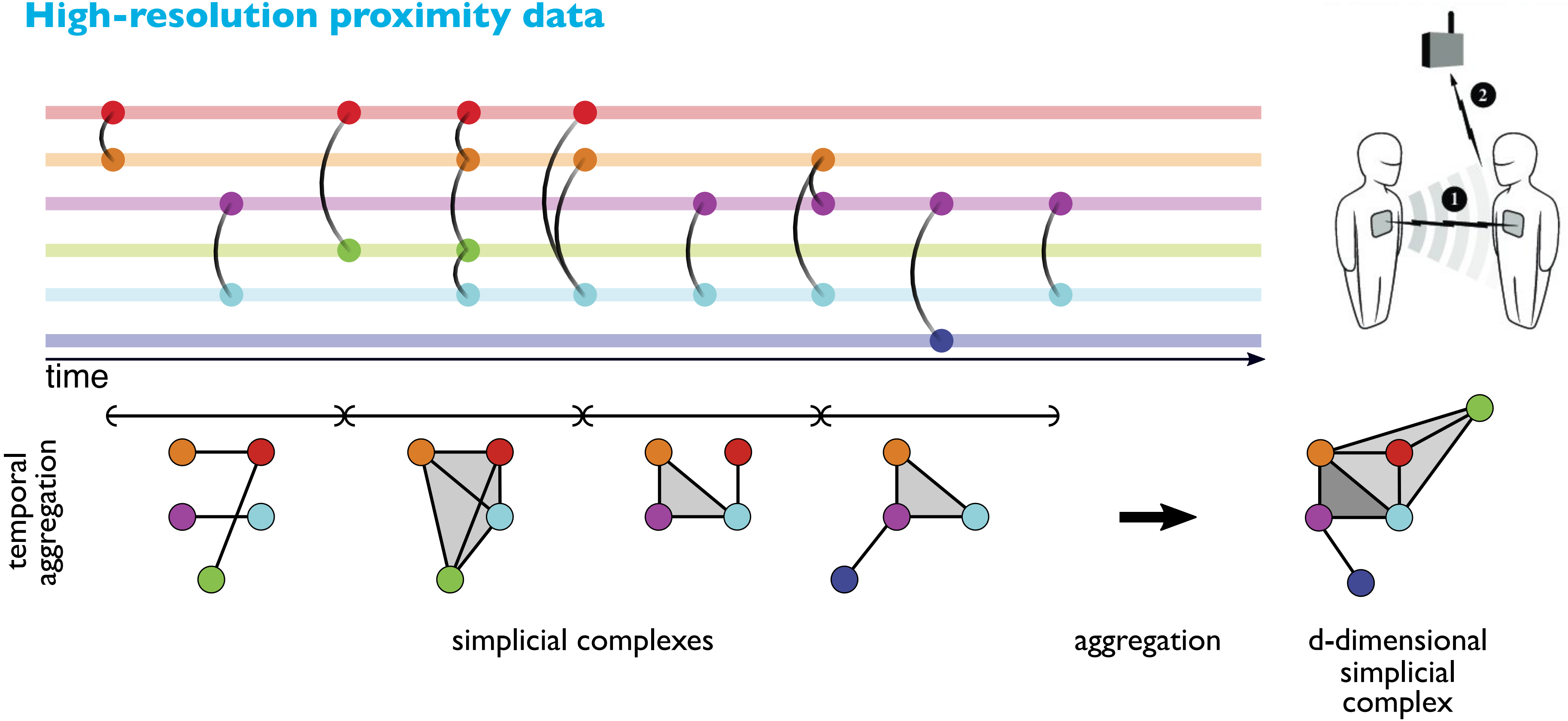

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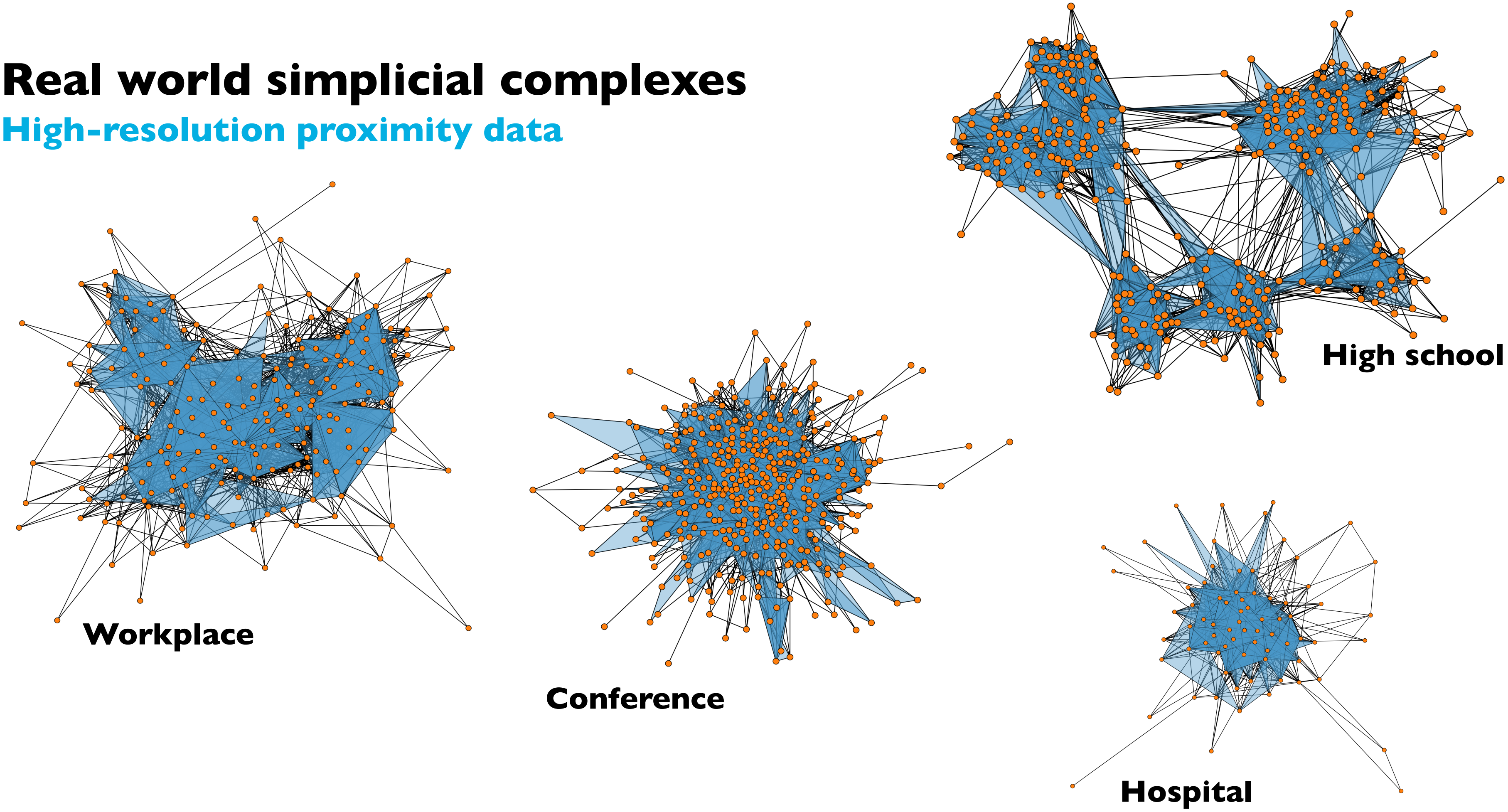


SocioPatterns M. Génois and A. Barrat, EPJ Data Science (2018)

Real world simplicial complexes

High-resolution proximity data

To reduce finite size effects

“Augmented”
Simplicial
Complex

Duplication of the lists of
sizes of the maximal
simplifies and simplicial
degrees of nodes

Real world simplicial complexes

High-resolution proximity data

To reduce finite size effects

"Augmented" Simplicial Complex

Duplication of the lists of sizes of the maximal simplifies and simplicial degrees of nodes

PHYSICAL REVIEW E **96**, 032312 (2017)

Construction of and efficient sampling from the simplicial configuration model

Jean-Gabriel Young,[1,*] Giovanni Petri,[2] Francesco Vaccarino,[2,3] and Alice Patania[2,3,†]

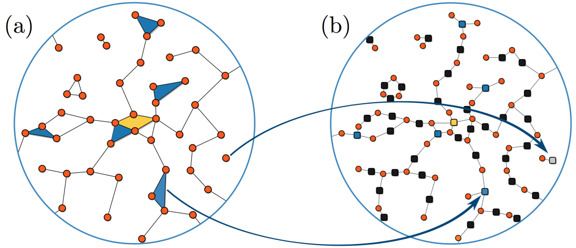
[1] *Département de Physique, de Génie Physique, et d'Optique, Université Laval, G1V 0A6 Québec (Québec), Canada*

[2] *ISI Foundation, 10126 Torino, Italy*

[3] *Dipartimento di Scienze Matematiche, Politecnico di Torino, 10129 Torino, Italy*

(Received 29 May 2017; published 22 September 2017)

Simplicial complexes are now a popular alternative to networks when it comes to describing the structure of complex systems, primarily because they encode multinode interactions explicitly. With this new description comes the need for principled null models that allow for easy comparison with empirical data. We propose a natural candidate, the *simplicial configuration model*. The core of our contribution is an efficient and uniform Markov chain Monte Carlo sampler for this model. We demonstrate its usefulness in a short case study by investigating the topology of three real systems and their randomized counterparts (using their Betti numbers). For two out of three systems, the model allows us to reject the hypothesis that there is no organization beyond the local scale.



Simplicial Configuration Model

Real world simplicial complexes

High-resolution proximity data

To reduce finite size effects

“Augmented” Simplicial Complex

Duplication of the lists of sizes of the maximal simplifies and simplicial degrees of nodes

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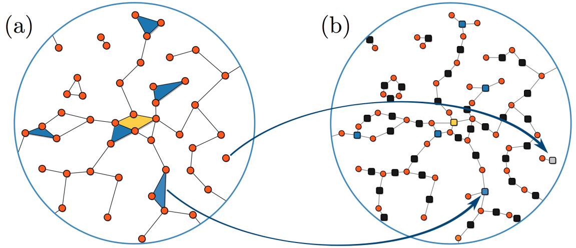


Simplicial Configuration Model

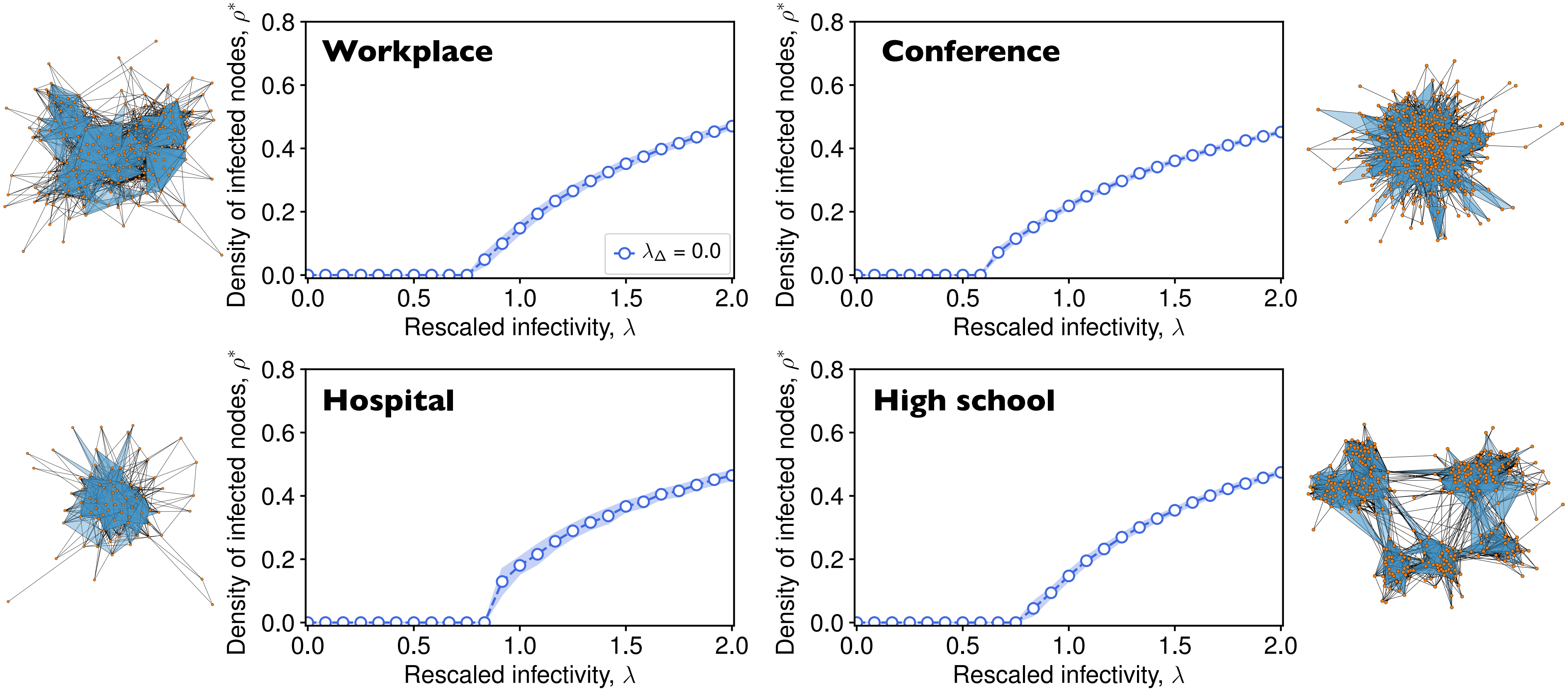
Processed Simplicial Complex

Same statistical properties of the input complex but of significantly larger size

Results

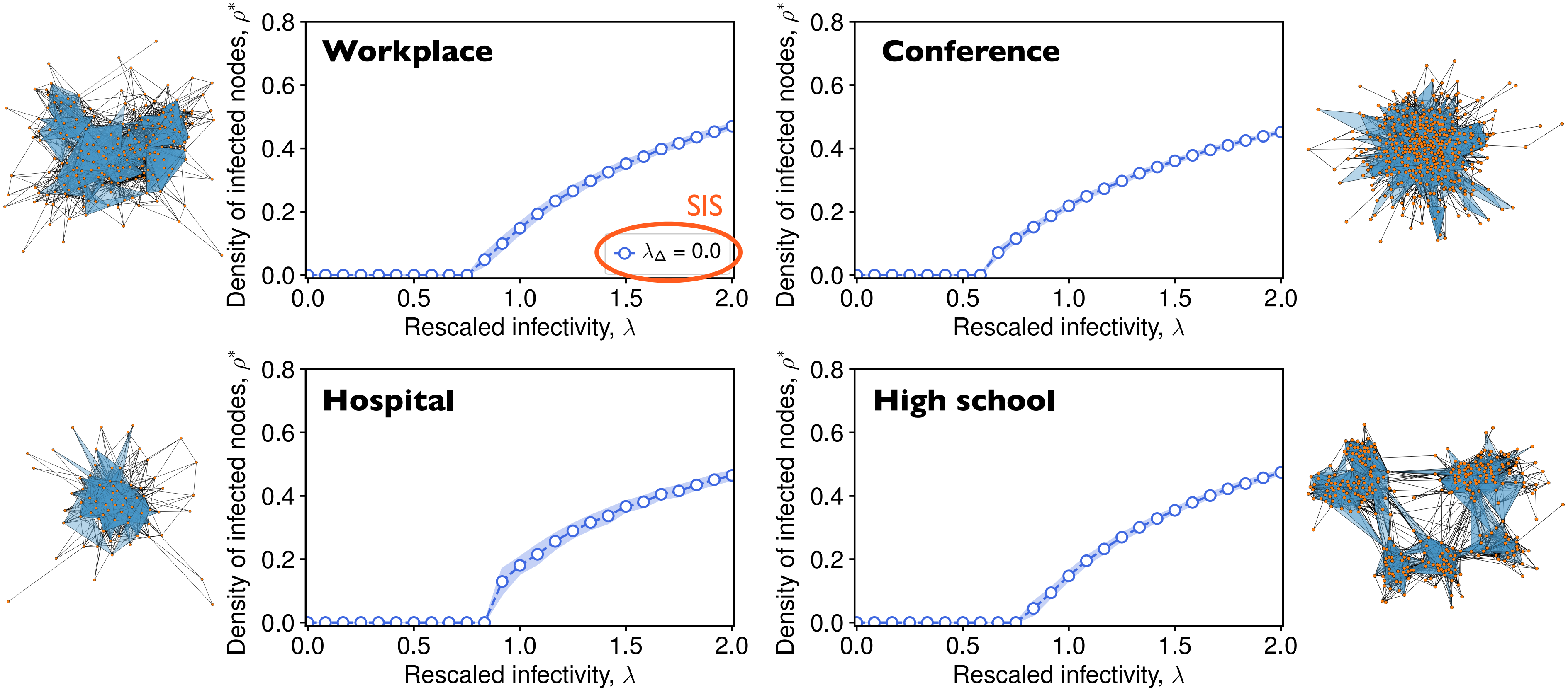
Results

High-resolution proximity data



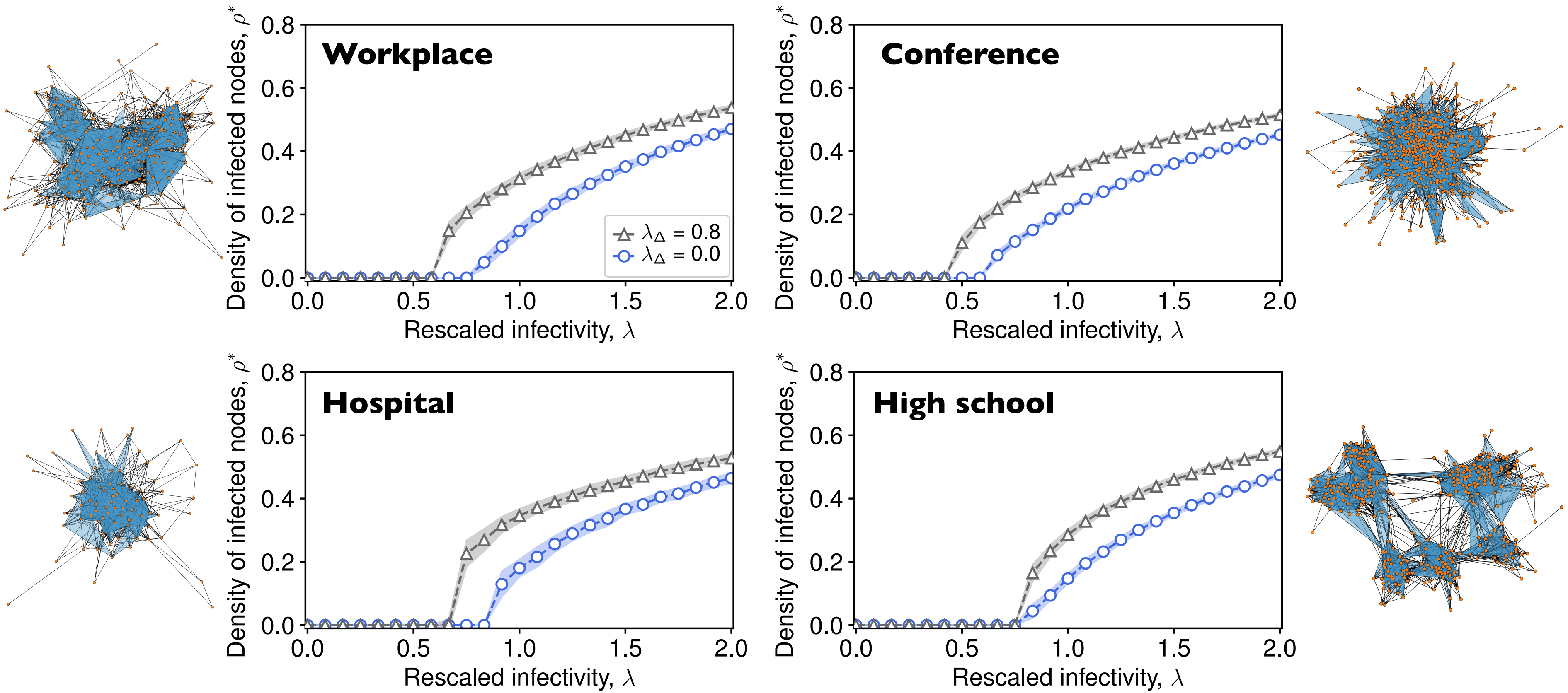
Results

High-resolution proximity data



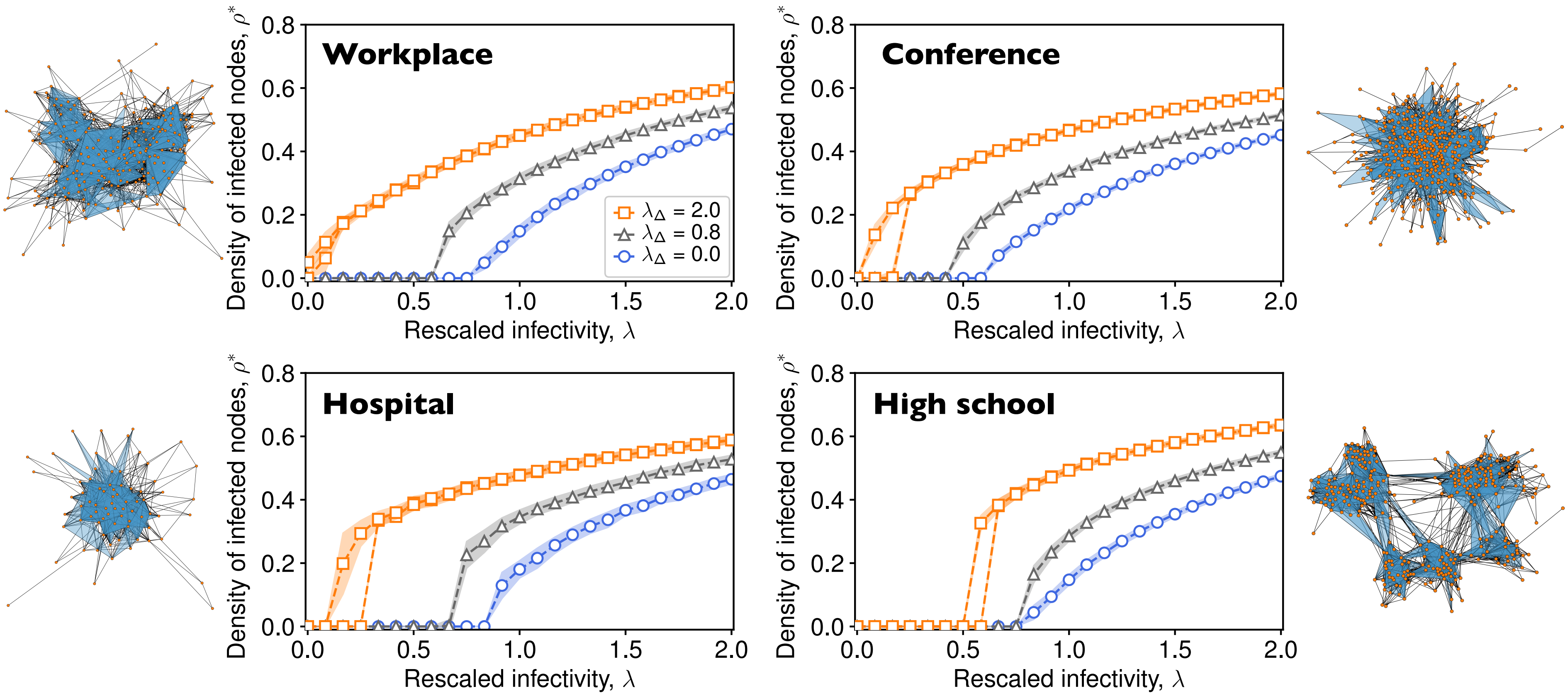
Results

High-resolution proximity data



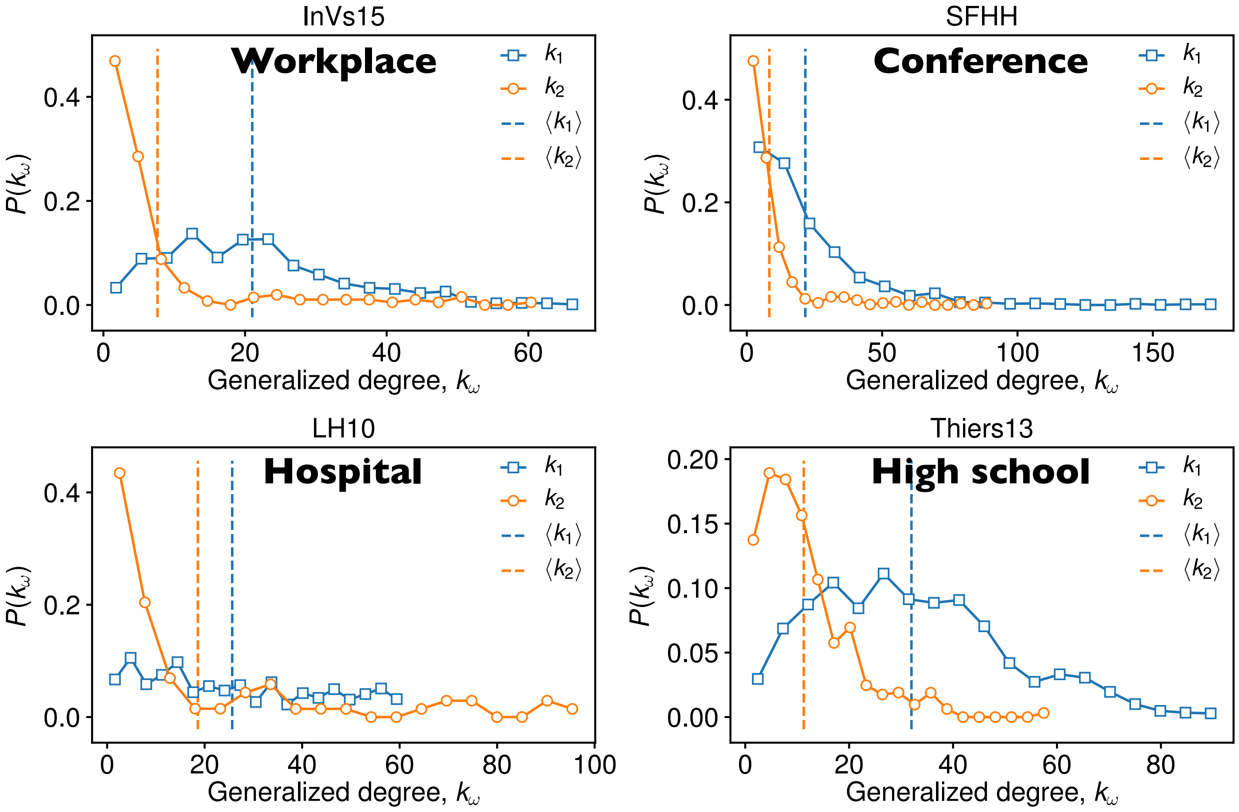
Results

High-resolution proximity data



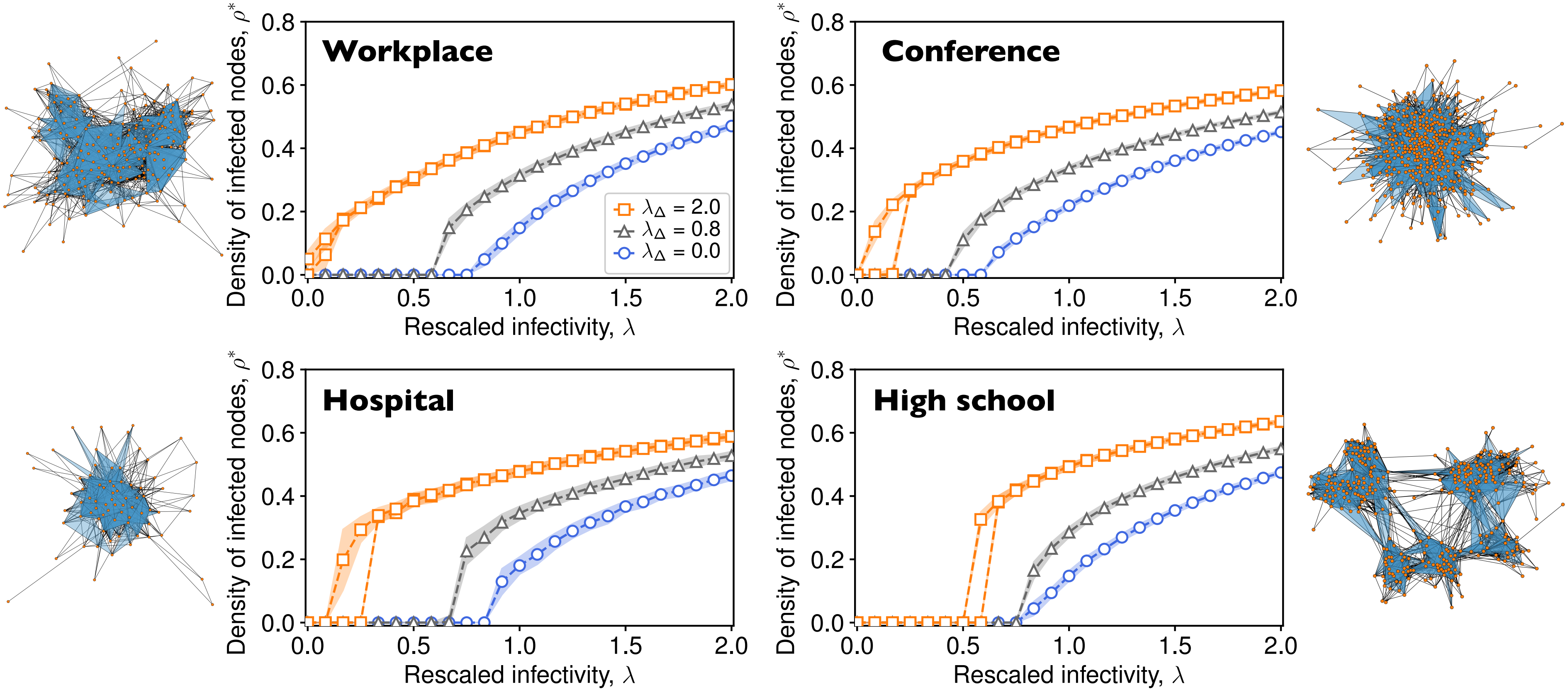
Empirical simplicial complexes

Generalised degree distributions



Results

High-resolution proximity data



Random Model

RSC Model

ER-like Random Simplicial Complexes

- Set of N vertices (0-simplices)
- Set of probabilities $\{p_1, \dots, p_k, \dots, p_D\}, \; p_k \in [0, 1]$

RSC Model

ER-like Random Simplicial Complexes

- Set of N vertices (0-simplices)
- Set of probabilities $\{p_1, \ldots, p_k, \ldots, p_D\}, \;\; p_k \in [0,1]$

Case D=2

$$(N, p_1, p_2)$$

RSC Model

ER-like Random Simplicial Complexes

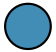
- Set of N vertices (0-simplices)
- Set of probabilities $\{p_1, \ldots, p_k, \ldots, p_D\}, \; p_k \in [0, 1]$

Case D=2

N

Number of vertices

(N, p_1, p_2)



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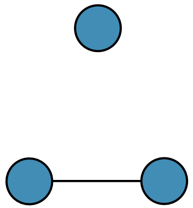
Case D=2

(N, p_1, p_2) $\rightarrow$

N
Number of vertices

p_1
probability of creating a simplex (i, j)



RSC Model

ER-like Random Simplicial Complexes

- Set of N vertices (0-simplices)
- Set of probabilities $\{p_1, \ldots, p_k, \ldots, p_D\}, \ p_k \in [0, 1]$

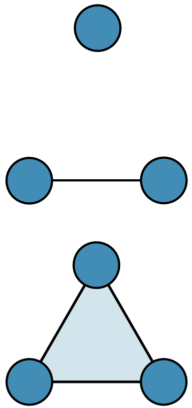
Case D=2

(N, p_1, p_2)

N
Number of vertices

p_1
probability of creating a simplex (i, j)

$p_2 \equiv p_\Delta$
probability of creating a simplex (i, j, k)



RSC Model

ER-like Random Simplicial Complexes

- Set of N vertices (0-simplices)
- Set of probabilities $\{p_1, \ldots, p_k, \ldots, p_D\}$, $p_k \in [0, 1]$

Case D=2

$$(N, p_1, p_2) \longrightarrow \boxed{\begin{aligned} \langle k \rangle &\approx (N-1)p_1 + 2\langle k_\Delta \rangle (1 - p_1) \\ \langle k_\Delta \rangle &= \frac{(N-1)(N-2)p_\Delta}{2} \end{aligned}}$$

RSC Model

ER-like Random Simplicial Complexes

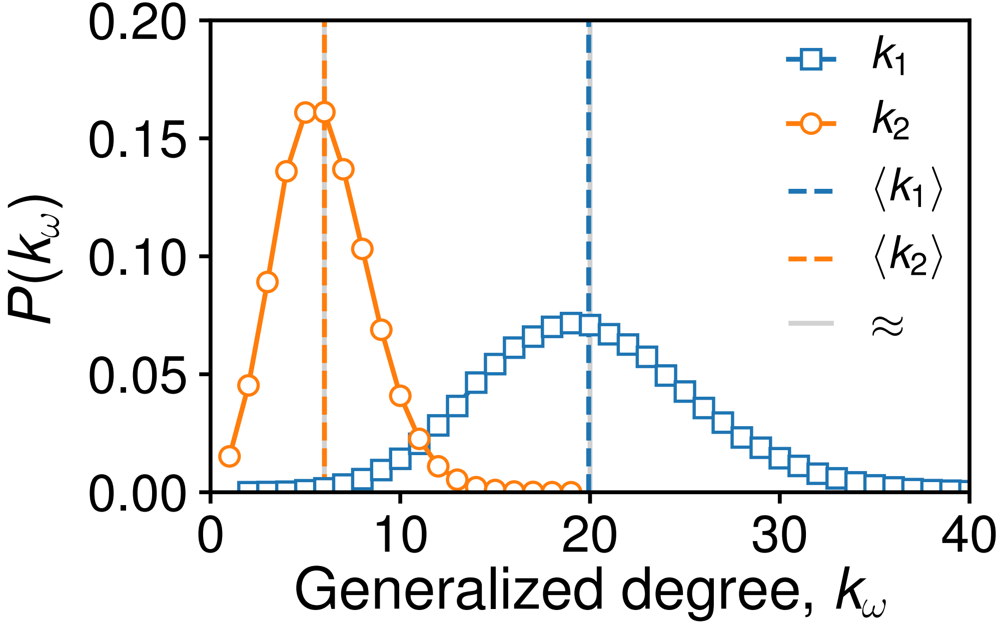
- Set of N vertices (0-simplices)
- Set of probabilities $\{p_1, \dots, p_k, \dots, p_D\}, \; p_k \in [0, 1]$

Case D=2

$$(N, p_1, p_2) \longrightarrow \boxed{\begin{aligned} p_1 &= \frac{\langle k \rangle - 2\langle k_\Delta \rangle}{(N-1) - 2\langle k_\Delta \rangle} \\ p_\Delta &= \frac{2\langle k_\Delta \rangle}{(N-1)(N-2)} \end{aligned}} \longrightarrow (N, \langle k \rangle, \langle k_\Delta \rangle)$$

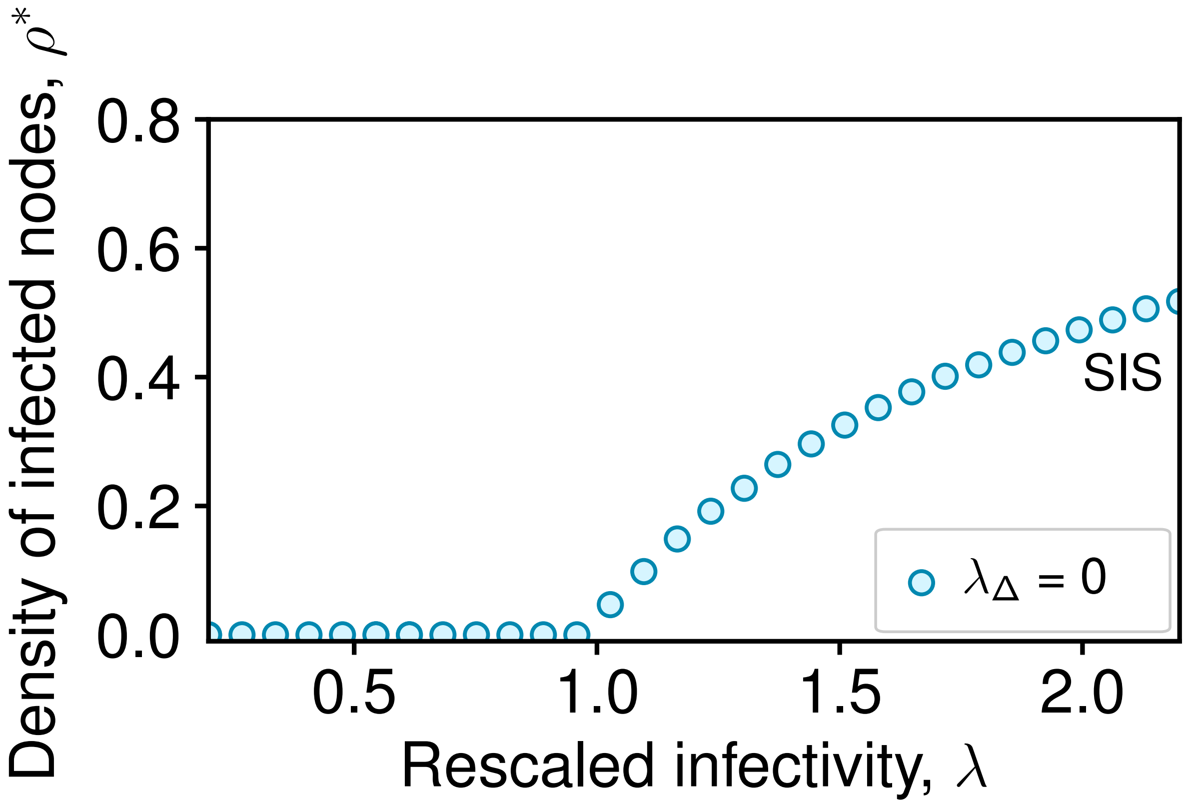
RSC Model

Generalised degree distribution



Results

Random Simplicial Complexes

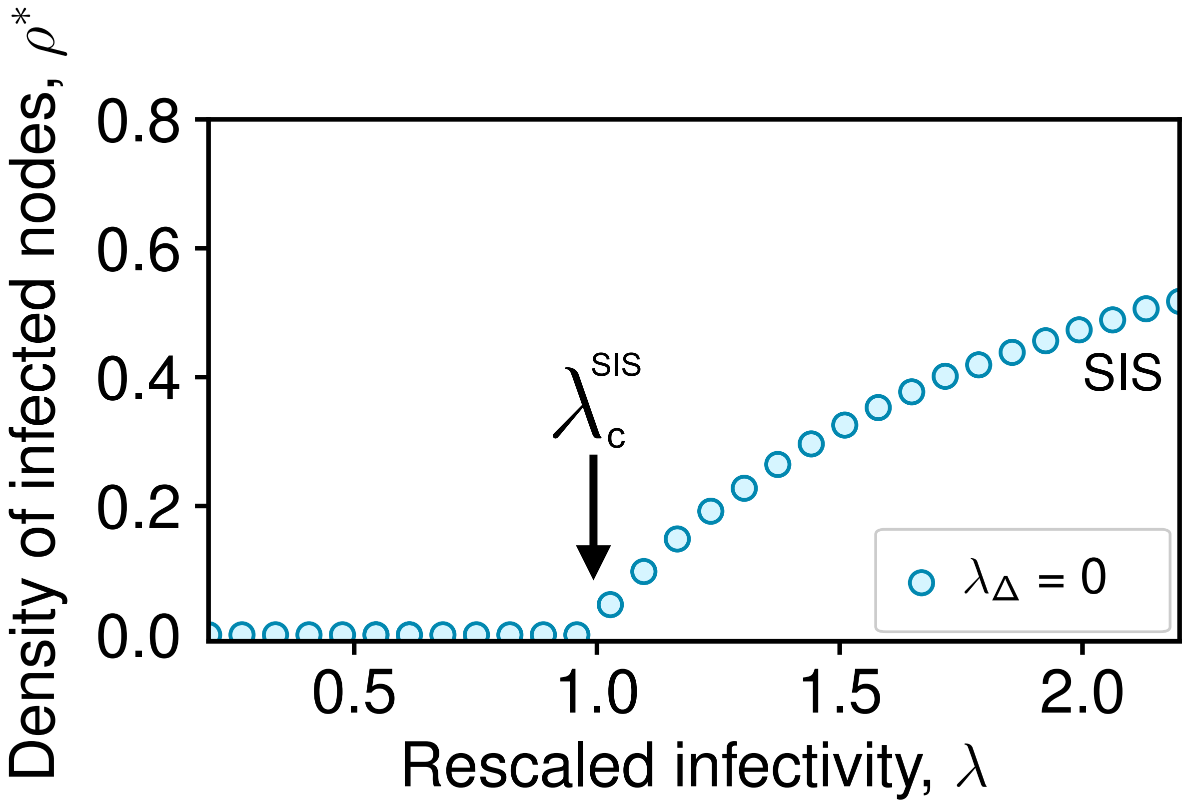


2-simplex infectivity

1-simplex infectivity

Results

Random Simplicial Complexes



2-simplex infectivity

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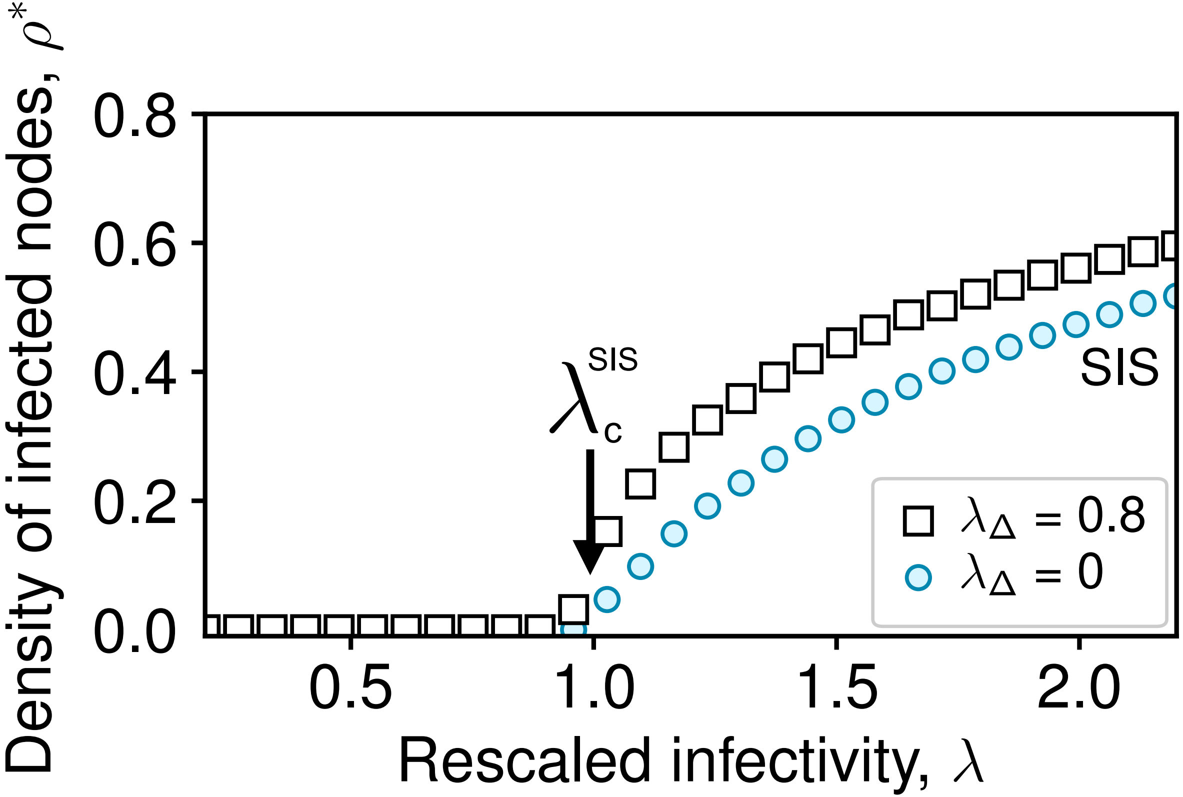
Random Simplicial Complexes



2-simplex infectivity

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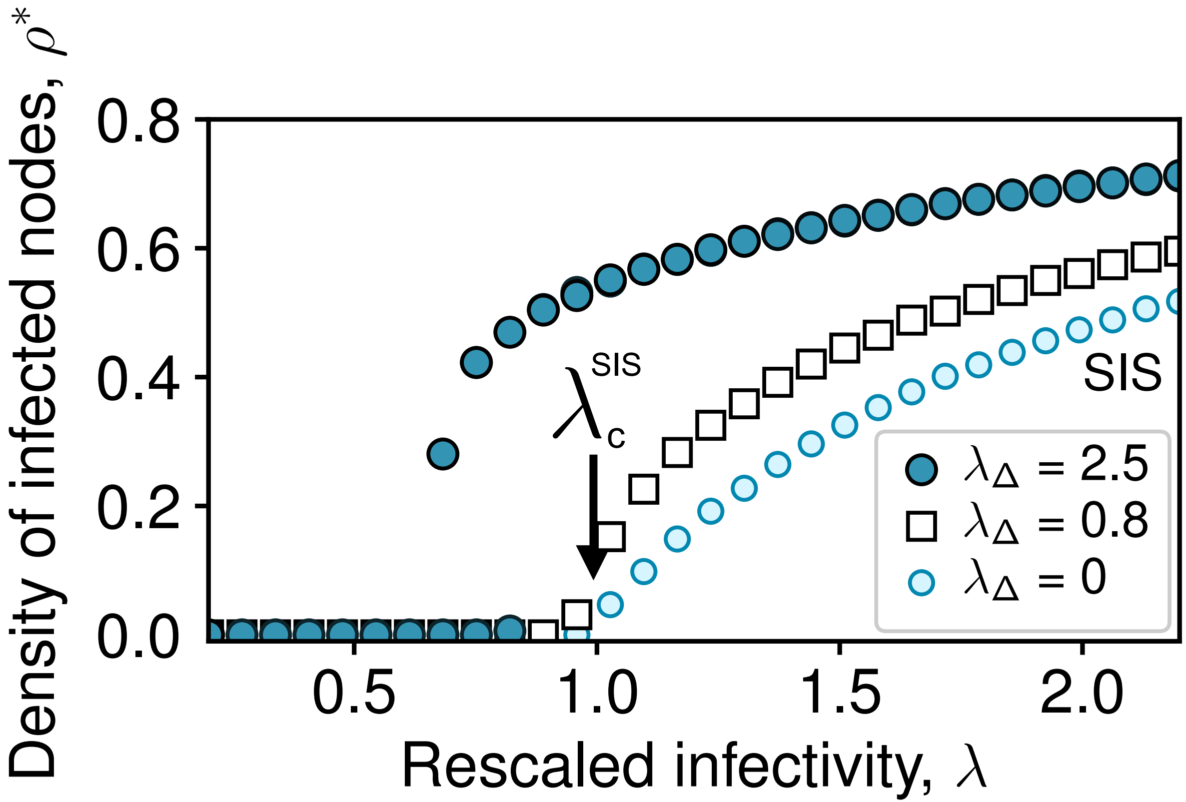
Results

Random Simplicial Complexes



2-simplex infectivity

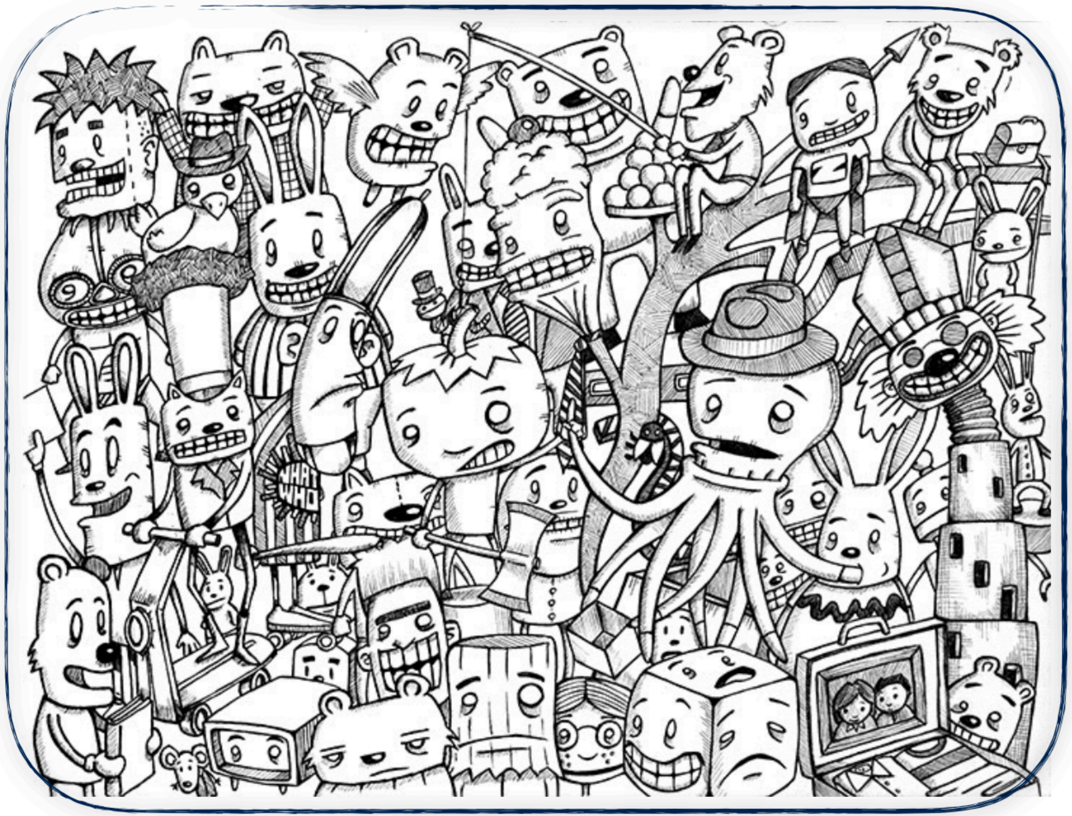
1-simplex infectivity

Mean Field Approach

Mean Field approach

Homogeneous mixing hypothesis

- All individuals are the same and behave equally
- Same number of contacts: network of contacts has very small degree fluctuations
- Timescale of infection faster than demographics (closed population)



Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$:

D=2

$$d_t\rho(t)$$

Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$:

D=2 loss of infectiousness

$$d_t\rho(t) = -\mu\rho(t)$$

Mean Field approach

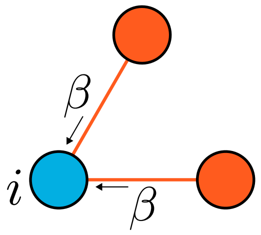
Temporal evolution of the density of infected nodes $\rho(t)$:

D=2

loss of infectiousness

$$d_t\rho(t) = \underbrace{-\mu\rho(t)}_{} + \underbrace{\beta\langle k\rangle\rho(t)\left[1-\rho(t)\right]}_{\text{new infections from 1-simplices}}$$

new infections
from 1-simplices



Mean Field approach

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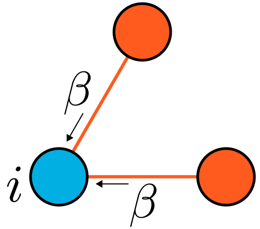
D=2

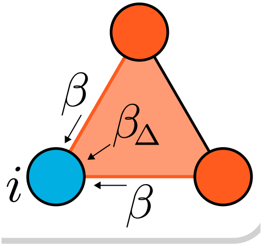
loss of infectiousness

$$d_t\rho(t) = -\mu\rho(t) + \beta\langle k\rangle\rho(t)\left[1-\rho(t)\right] + \beta_\Delta\langle k_\Delta\rangle\rho^2(t)\left[1-\rho(t)\right]$$

new infections
from 1-simplices

new infections
from 2-simplices





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Set of infection probabilities $B \equiv \{\beta_\omega, \omega = 1, \cdots, D\}$

D

$$d_t\rho(t) = \underbrace{-\mu\rho(t)}_{\text{loss of infectiousness}} + \sum_{\omega=1}^{D} \underbrace{\beta_\omega\langle k_\omega\rangle\rho^\omega(t)\left[1-\rho(t)\right]}_{\text{new infections}}$$

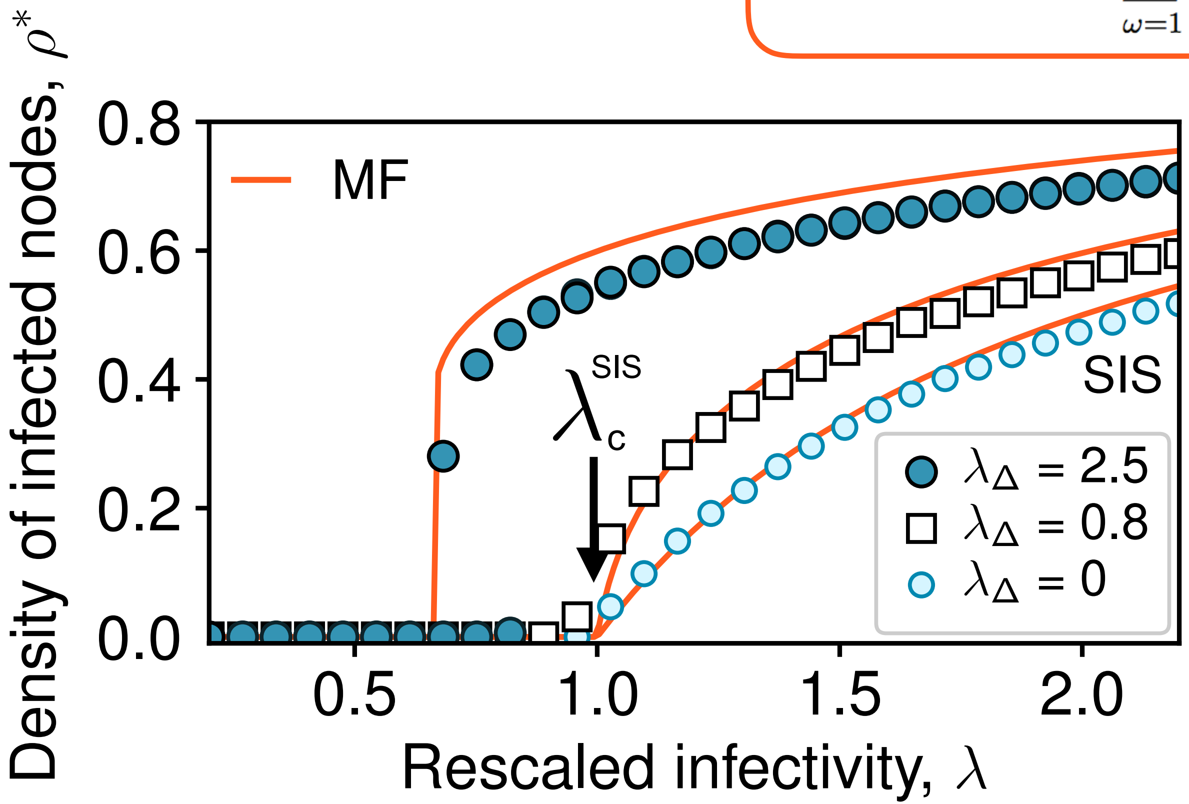
Results

Random Simplicial Complexes

Mean Field

$$d_t\rho(t) = -\mu\rho(t) + \sum_{\omega=1}^{D} \beta_\omega \langle k_\omega\rangle \rho^\omega(t)\left[1-\rho(t)\right]$$

D=2



2-simplex infectivity

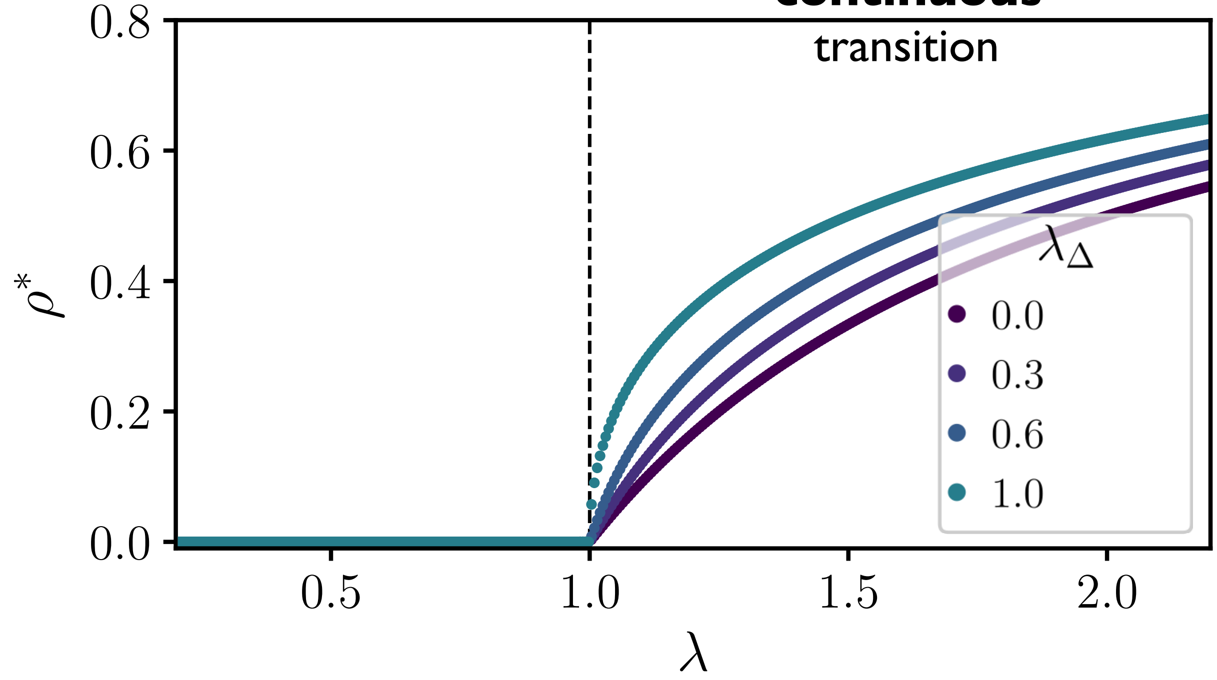
1-simplex infectivity

Mean Field approach

$\lambda_\Delta \leqslant 1$

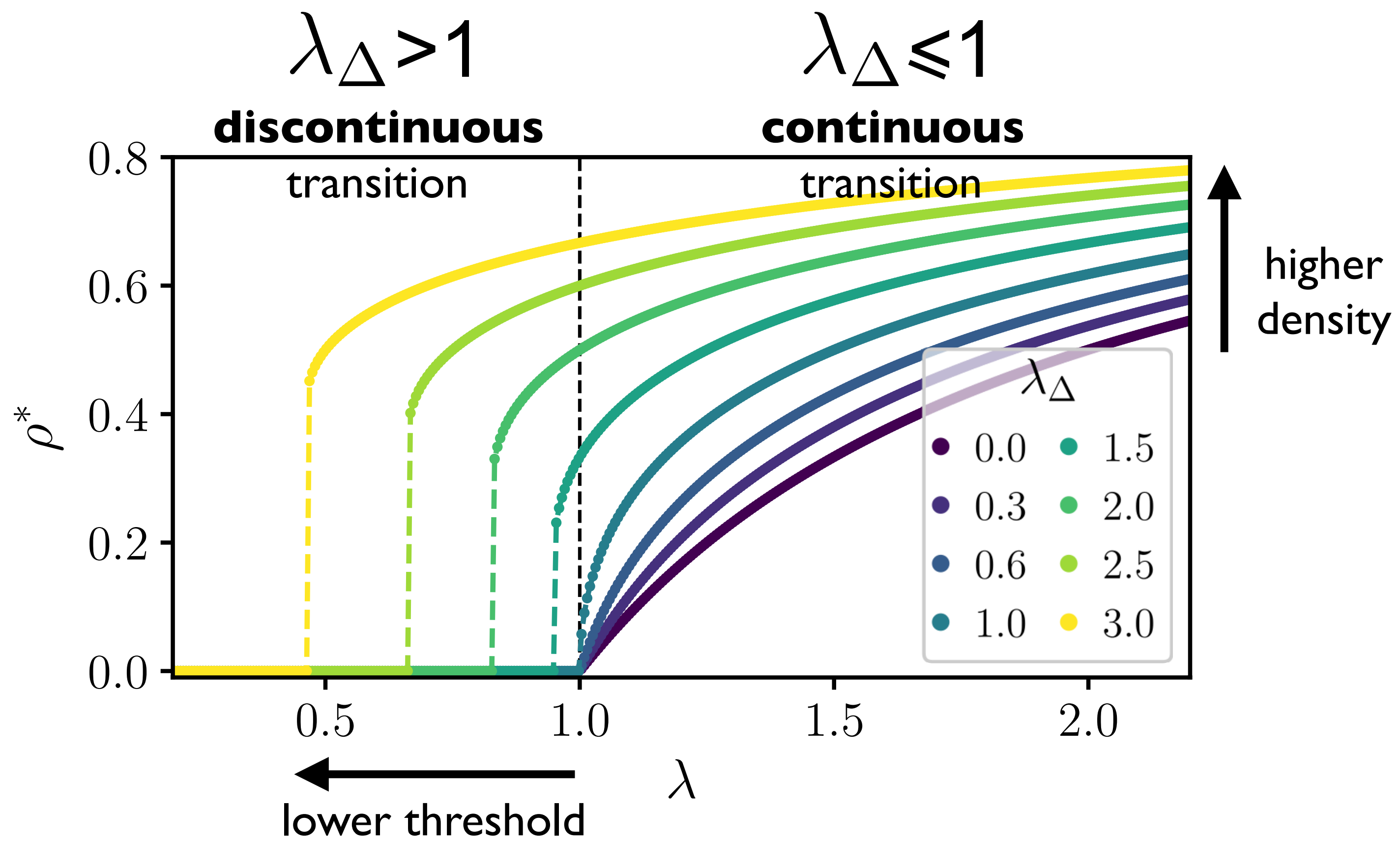
continuous

transition



higher density

Mean Field approach



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$$d_t\rho(t) = -\rho(t)(\rho(t)-\rho^*_{2+})(\rho(t)-\rho^*_{2-})$$

Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$:

D=2

loss of infectiousness

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new infections
from 1-simplices

new infections
from 2-simplices

$$d_t\rho(t) = -\rho(t)(\rho(t)-\rho^*_{2+})(\rho(t)-\rho^*_{2-})$$

The **steady state** equation $d_t\rho(t) = 0$ has up to three solution in the acceptable range $\rho \in [0,1]$

Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$:

D=2 loss of infectiousness

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new infections from 1-simplices

new infections from 2-simplices

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The **steady state** equation $d_t\rho(t) = 0$ has up to three solution in the acceptable range $\rho \in [0,1]$

$$\rho^*_1 = 0$$

absorbing epidemic-free state

Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$:

D=2

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$$d_t\rho(t) = -\mu\rho(t) + \underbrace{\beta\langle k\rangle\rho(t)\left[1-\rho(t)\right]}_{\text{new infections from 1-simplices}} + \underbrace{\beta_\Delta\langle k_\Delta\rangle\rho^2(t)\left[1-\rho(t)\right]}_{\text{new infections from 2-simplices}}$$

$$d_t\rho(t) = -\rho(t)(\rho(t) - \rho^*_{2+})(\rho(t) - \rho^*_{2-})$$

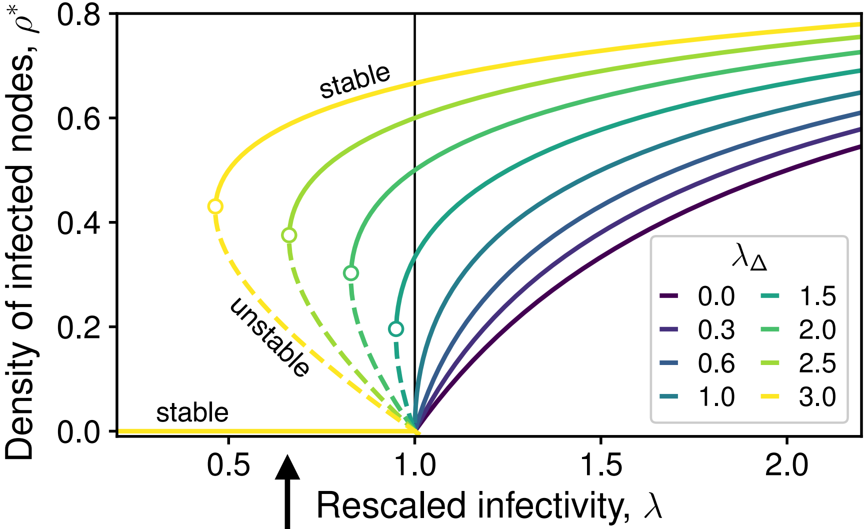
The **steady state** equation $d_t\rho(t) = 0$ has up to three solution in the acceptable range $\rho \in [0,1]$

$$\rho^*_1 = 0$$

absorbing epidemic-free state

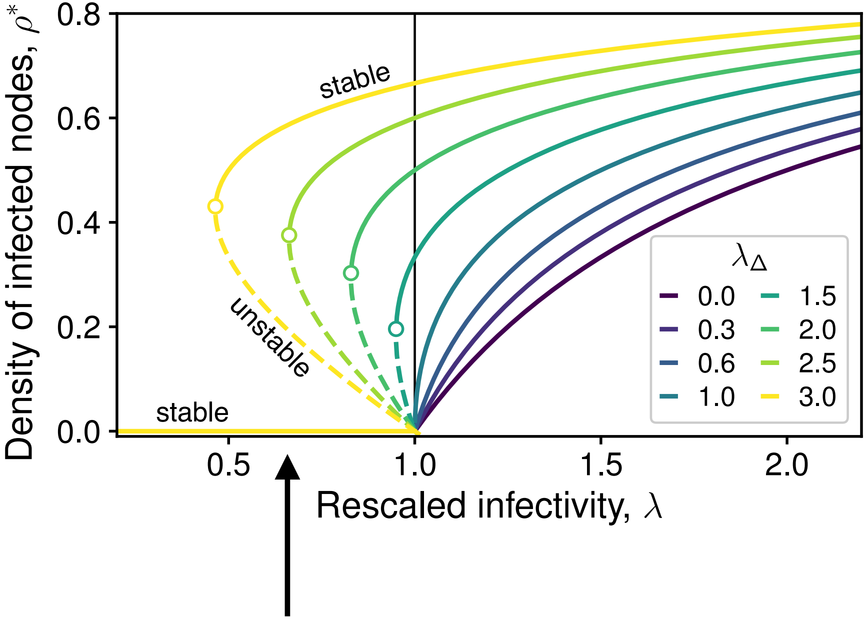
$$\rho^*_{2\pm} = \frac{\lambda_\Delta - \lambda \pm \sqrt{(\lambda - \lambda_\Delta)^2 - 4\lambda_\Delta(1-\lambda)}}{2\lambda_\Delta}$$

Mean Field approach

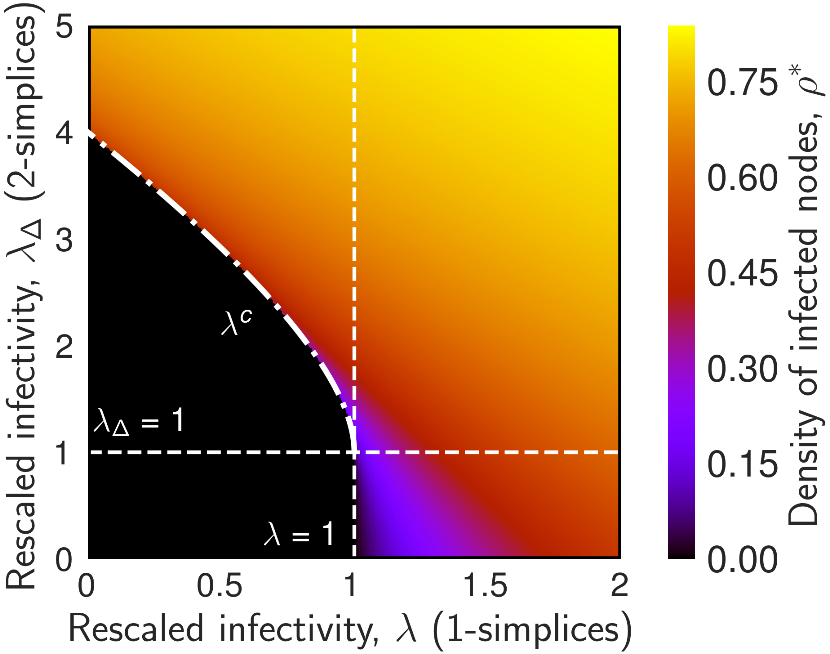


Bistable region

Mean Field approach

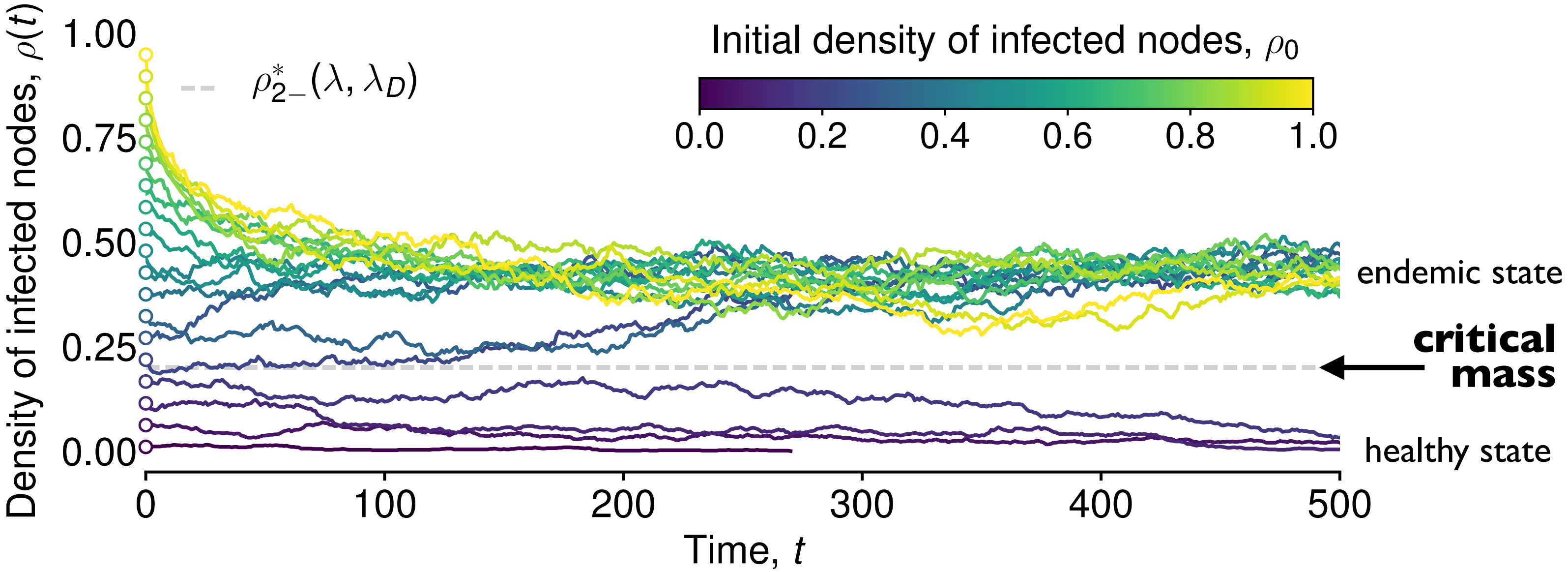


Bistable region



Mean Field approach

Dependency on initial conditions



Summing up - SIMPLAGION

- Considering **high-order** interactions in social contagion processes
  - Social structure modelled as a **simplicial complex**
  - Contagion occurs in **group interactions** (with different transmission rates)

Summing up - SIMPLAGION

- Considering **high-order** interactions in social contagion processes
  - Social structure modelled as a **simplicial complex**
  - Contagion occurs in **group interactions** (with different transmission rates)
- Numerical simulations on both **empirical** and **synthetic** simplicial complexes
  - **Discontinuous** transition
  - Dependence on the size of the **seed** (critical mass)

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  - Simplagion on more general simplicial complexes - HMF
  - Inference from structural + dynamical data
  - Extension to other dynamical processes - Simplicial Kuramoto model

Actually...

$$\dot{\theta}_i = \omega_i + \frac{K}{N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \sin\left(\theta_j + \theta_k - 2\theta_i\right)$$

Abrupt Desynchronization and Extensive Multistability in Globally Coupled Oscillator Simplices

Per Sebastian Skardal[1,*] and Alex Arenas[2]

[1]*Department of Mathematics, Trinity College, Hartford, CT 06106, USA*
[2]*Departament d'Enginyeria Informatica i Matemátiques, Universitat Rovira i Virgili, 43007 Tarragona, Spain*

Collective behavior in large ensembles of dynamical units with non-pairwise interactions may play an important role in several systems ranging from brain function to social networks. Despite recent work pointing to simplicial structure, i.e., higher-order interactions between three or more units at a time, their dynamical characteristics remain poorly understood. Here we present an analysis of the collective dynamics of such a simplicial system, namely coupled phase oscillators with three-way interactions. The simplicial structure gives rise to a number of novel phenomena, most notably a continuum of abrupt desynchronization transitions with no abrupt synchronization transition counterpart, as well as, extensive multistability whereby infinitely many stable partially synchronized states exist. Our analysis sheds light on the complexity that can arise in physical systems with simplicial interactions like the human brain and the role that simplicial interactions play in storing information.

- Future work:
  - Simplagion on more general simplicial complexes - HMF
  - Inference from structural + dynamical data
  - Extension to other dynamical processes - Simplicial Kuramoto model?

Check out the arXiv!

Simplicial models of social contagion

Iacopo Iacopini,[1,*] Giovanni Petri,[2,3] Alain Barrat,[4,2] and Vito Latora[1,5,6,7]

[1] *School of Mathematical Sciences, Queen Mary University of London, London E1 4NS, United Kingdom*
[2] *ISI Foundation, Via Chisola 5, 10126 Turin, Italy*
[3] *ISI Global Science Foundation,33 W 42nd St10036 New York NY, United States*
[4] *Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France*
[5] *The Alan Turing Institute, The British Library, London NW1 2DB, UK*
[6] *Dipartimento di Fisica ed Astronomia, Università di Catania and INFN, I-95123 Catania, Italy*
[7] *Complexity Science Hub Vienna (CSHV), Vienna, Austria*

Complex networks have been successfully used to describe the spreading of a disease between the individuals of a population. Conversely, pairwise interactions are often not enough to characterize processes of social contagion, such as opinion formation or the adoption of novelties, where a more complex dynamics of peer influence and reinforcement mechanisms is at work. We introduce here a high-order model of social contagion in which a social system is represented by a simplicial complex and the contagion can occur, with different transmission rates, over the links or through interactions in groups of different sizes. Numerical simulations of the model on synthetic simplicial complexes and analytical results highlight the emergence of novel phenomena, such as the appearance of an explosive transition induced by the high-order interactions.

Iacopo Iacopini

i.iacopini@qmul.ac.uk

@iacopoiacopini

and thanks to…

Giovanni Petri
(ISI Foundation)

Alain Barrat
(CNRS)

Vito Latora
(QMUL)

Backup

Sociopatterns data sets

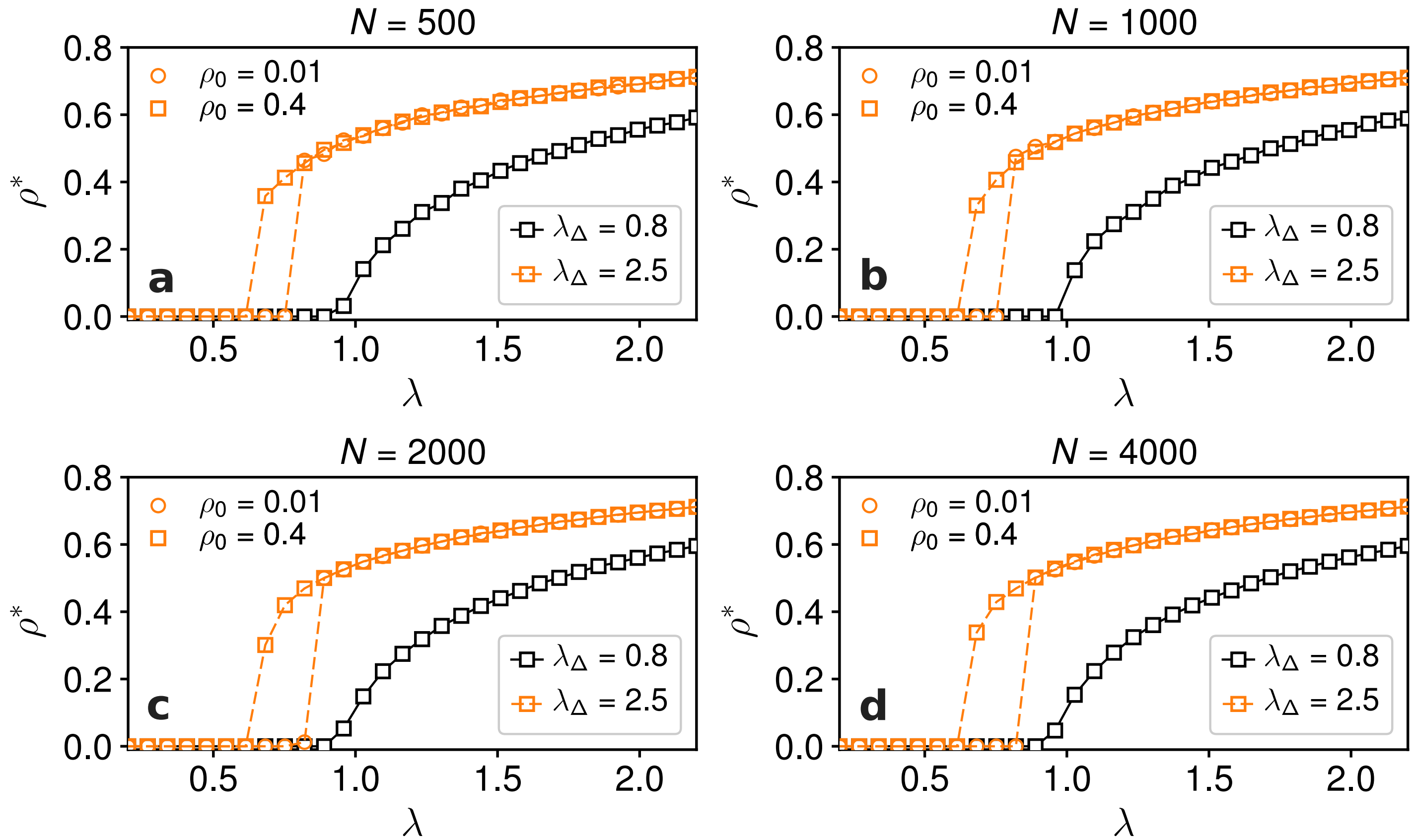
Dataset	Context	$\langle k \rangle$	$\langle k_\Delta \rangle$	$\langle k \rangle^{\text{aug}}$	$\langle k_\Delta \rangle^{\text{aug}}$
InVS15	Workplace	16.9	7.0	21.0	7.0
SFHH	Conference	15.0	7.6	21.6	7.7
LH10	Hospital	19.1	17.1	25.7	17.5
Thiers13	High school	20.1	10.9	32.0	11.1

Table 1: Average generalized degree of the four real-world simplicial complexes constructed from the considered data sets (before and after the data augmentation).

SocioPatterns

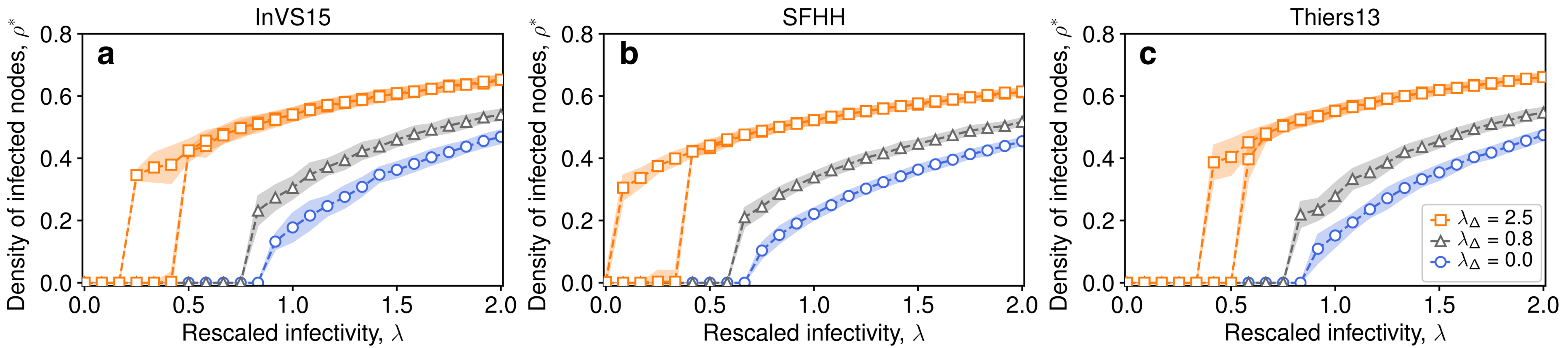
Simplagion on RSC

Size effects



Simplagion on empirical simplicial complexes

Without data augmentation



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